disropt Documentation

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disropt is a Python package developed within the excellence research program ERC in the project OPT4SMART. The aim of this package is to provide an easy way to run distributed optimization algorithms that can be executed by a network of peer copmuting systems.

A comprehensive guide to **disropt** can be found in the *Tutorial*. Many examples are provided in the *Examples* section, while the *API Documentation* can be checked for more details. The package is equipped with some commonly used objective functions and constraints which can be directly used.

disropt currently supports MPI in order to emulate peer-to-peer communication. However, custom communication protocols can be also implemented.

For example, the following Python code generates an unconstrained, quadratic optimization problem with a 5-dimensional decision variable, which is solved using the so-called *Gradient Tracking*.

Listing 1: basic_example.py

```
import numpy as np
from disropt.agents import Agent
from disropt.algorithms.gradient_tracking import GradientTracking
from disropt.functions import QuadraticForm, Variable
from disropt.utils.graph_constructor import MPIgraph
from disropt.problems import Problem

# generate communication graph (everyone uses the same seed)
comm_graph = MPIgraph('random_binomial', 'metropolis')
agent_id, in_nbrs, out_nbrs, in_weights, _ = comm_graph.get_local_info()

# size of optimization variable
n = 5

# generate quadratic cost function
np.random.seed()
Q = np.random.randn(n, n)
Q = Q.transpose() @ Q
x = Variable(n)
func = QuadraticForm(x - np.random.randn(n, 1), Q)

# create Problem and Agent
agent = Agent(in_nbrs, out_nbrs, in_weights=in_weights)
agent.set_problem(Problem(func))

# run the algorithm
x0 = np.random.rand(n, 1)
algorithm = GradientTracking(agent, x0)
algorithm.run(iterations=1000, stepsize=0.01)

print("Agent {} - solution estimate: {}".format(agent_id, algorithm.get_result().
↪flatten()))
```

This code can be executed over a network with 8 agents by issuing the following command:

```
mpirun -np 8 python example.py
```

CHAPTER

ONE

INSTALLATION

The **disropt** package supports Python 3 and can be installed through pip by running in your terminal:

```
pip install disropt
```

An MPI implementation needs to be installed on you platform in order to use **disropt**.

CHAPTER

TWO

QUICK START

For the installation of the package, refer to the *Installation* section.

To run an algorithm, it suffices to create an instance of the corresponding class and then call the method `run()`. The class constructor requires an instance of the *Agent* class, which must contain the local information available to the agent to run the algorithm.

2.1 Example with the consensus algorithm

For example, to run the *Consensus* algorithm, first create an instance of the Agent class with the graph information:

```
agent = Agent(in_neighbors, out_neighbors, in_weights)
```

where the variables `in_neighbors`, `out_neighbors` and `in_weights` are previously initialized lists. Then, create an instance of the Consensus class with the agent's initial condition and call the method `run()`:

```
algorithm = Consensus(agent=agent, initial_condition=x0)
algorithm.run(iterations=100)
```

The method `get_result()` can be called to get the output of the algorithm:

```
print("Output of agent {}: {}".format(agent.id, algorithm.get_result()))
```

All the code showed so far is python code and must be enclosed in a script file. To actually run the code with MPI (which is the default *Communicator*), run on a terminal:

```
mpirun -np 8 python script.py
```

where in this case the script file `script.py` is executed over 8 processors.

2.2 Example with distributed optimization

For distributed optimization algorithms, the workflow is almost the same, except that the Agent class must be equipped with the problem data that is locally available to the agent. The problem data should be passed as an instance of the *Problem* class (or one of its children) *before* creating the instance of the algorithm class.

For example, to run the *Distributed subgradient* algorithm, the cost function must be passed to the instance of the Agent class after its initialization:

```
problem = Problem(objective_function)
agent.set_problem(problem)
```

where the variable `objective_function` is the agent's objective function in the cost-coupled problem.

Then, the algorithm can be run just like in the Consensus case:

```
algorithm = SubgradientMethod(agent=agent, initial_condition=x0)
algorithm.run(iterations=100)
print("Output of agent {}: {}".format(agent.id, algorithm.get_result()))
```

and on the terminal:

```
mpirun -np 8 python script.py
```

CHAPTER

THREE

TUTORIAL

disropt is a Python package for distributed optimization over peer-to-peer networks of computing units called agents. The main idea of distributed optimization is to solve an optimization problem (enjoying a given structure) over a (possibly unstructured) network of processors. Each agent can perform local computation and can exchange information with only its neighbors in the network. A distributed algorithm consists of an iterative procedure in which each agent maintains a local estimate of the problem solution which is properly updated in order to converge towards the solution.

Formally, an optimization problem is a mathematical problem which consists in finding a minimum of a function while satisfying a given set of constraints. In symbols,

$$\begin{aligned} &\min_x \ f(x) \\ &\text{subject to } \ x \in X, \end{aligned}$$

where $x \in \mathbb{R}^d$ is called optimization variable, $f : \mathbb{R}^d \to \mathbb{R}.$ is called cost function and $X \subseteq \mathbb{R}^d$ describes the problem constraints. The optimization problem is assumed to be feasible and has finite optimal cost. Thus, it admits at least an optimal solution that is usually denoted as $x^\star$. The optimal solution is a vector that satisfies all the constraints and attains the optimal cost.

3.1 Distributed optimization set-ups

Distributed optimization problems usually arising in applications usually enjoy a proper structure in their mathematical formulation. In **disropt**, three different optimization set-ups are available. As for their solution, see the *Algorithms* page for a list of all the implemented distributed algorithms (classified by optimization set-up to which they apply).

3.1.1 Cost-coupled set-up

In this optimization set-up, the cost function is expressed as the sum of cost functions f_i and all of them depend on a common optimization variable x. Formally, the set-up is

$$\begin{aligned} &\min_x \ \sum_{i=1}^N f_i(x) \\ &\text{subject to } \ x \in X, \end{aligned}$$

where $x \in \mathbb{R}^d$ and $X \subseteq \mathbb{R}^d$. The global constraint set X is common to all agents, while $f_i : \mathbb{R}^d \to \mathbb{R}$ is assumed to be known by agent i only, for all $i \in \{1, \dots, N\}$.

In some applications, the constraint set X can be expressed as the intersection of local constraint sets, i.e.,

$$X = \bigcap_{i=1}^{N} X_i$$

where each $X_i \subseteq \mathbb{R}^d$ is meant to be known by agent i only, for all $i \in \{1, \dots, N\}$.

The goal for distributed algorithms for the cost-coupled set-up is that all agent estimates are eventually consensual to an optimal solution $x^\star$ of the problem.

3.1.2 Common cost set-up

In this optimization set-up, there is a unique cost function f that depends on a common optimization variable x, and the optimization variable must further satisfy local constraints. Formally, the set-up is

$$\begin{aligned} &\min_{x} \ f(x) \\ &\text{subject to } \ x \in \bigcap_{i=1}^{N} X_i, \end{aligned}$$

where $x \in \mathbb{R}^d$ and each $X_i \subseteq \mathbb{R}^d$. The cost function f is assumed to be known by all the agents, while each set X_i is assumed to be known by agent i only, for all $i \in \{1, \dots, N\}$.

The goal for distributed algorithms for the common-cost set-up is that all agent estimates are eventually consensual to an optimal solution $x^\star$ of the problem.

3.1.3 Constraint-coupled set-up

In this optimization set-up, the cost function is expressed as the sum of local cost functions f_i that depend on a local optimization variable x_i. The variables must satisfy local constraints (involving only each optimization variable x_i) and global coupling constraints (involving all the optimization variables). Formally, the set-up is

$$\begin{aligned} &\min_{x_1,\dots,x_N} \ \sum_{i=1}^{N} f_i(x_i) \\ &\text{subject to } \ x_i \in X_i, \qquad i \in \{1, \dots, N\} \\ &\qquad\qquad \sum_{i=1}^{N} g_i(x_i) \leq 0, \end{aligned}$$

where each $x_i \in \mathbb{R}^{d_i}$, $X_i \subseteq \mathbb{R}^{d_i}$, $f_i : \mathbb{R}^{d_i} \to \mathbb{R}$ and $g_i : \mathbb{R}^{d_i} \to \mathbb{R}^S$ for all $i \in \{1, \dots, N\}$. Here the symbol $\leq$ is also used to denote component-wise inequality for vectors. Therefore, the optimization variable consists of the stack of all x_i, namely the vector $(x_1, \dots, x_N)$. All the quantities with the index i are assumed to be known by agent i only, for all $i \in \{1, \dots, N\}$. The function g_i, with values in $\mathbb{R}^S$, is used to express the i-th contribution to S coupling constraints among all the variables.

The goal for distributed algorithms for the constraint-coupled set-up is that each agent estimate is asymptotically equal to its portion $x_i^\star \in X_i$ of an optimal solution $(x_1^\star, \dots, x_N^\star)$ of the problem.

3.2 Objective functions and constraints

3.2.1 Functions

disropt comes with many already implemented mathematical functions. Functions are defined in terms of optimization variables (`Variable`) or other functions. Let us start by defining a `Variable` object as:

```
from disropt.functions import Variable
n = 2 # dimension of the variable
x = Variable(n)
print(x.input_shape) # -> (2, 1)
print(x.output_shape) # -> (2, 1)
```

Now, suppose you want to define an affine function $f(x) = A^\top x - b$ with $A \in \mathbb{R}^{2\times 2}$ and $b \in \mathbb{R}^2$:

```
import numpy as np
a = 1
A = np.array([[1,2], [2,4]])
b = np.array([[1], [1]])
f = A @ x - b

# or, alternatively
from disropt.functions import AffineForm
f = AffineForm(x, A, b)
```

The composition of functions is fully supported. Suppose you want to define a function $g(x) = f(x)^\top Q f(x)$, then:

```
from disropt.functions import QuadraticForm
Q = np.random.rand(2,2)
g = QuadraticForm(f, Q) # or: g = f @ (Q.tranpose() @ f)
print(g.input_shape) # -> (2, 1)
print(g.output_shape) # -> (1, 1)
```

Currently supported operations with functions are sum (+), difference (-), product(*) and matrix product (@). Combination with numpy arrays is supported as well.

Function properties and methods

Each function has three properties that can be checked: differentiablity, being affine and quadratic:

```
g.is_differentiable # -> True
g.is_affine # -> False
g.is_quadratic # -> True
f.is_affine # -> True
```

and their input and output shapes can be obtained as

```
g.output_shape # -> (1,1)
g.input_shape # -> (2,1)
```

Moreover, it is possible to evaluate functions at desired points and to obtain the corresponding (sub)gradient/jacobian/hessian as:

```
pt = np.random.rand(2,1)
# the value of g computed at pt is obtained as
g.eval(pt)
# the value of the jacobian of g computed at pt is
g.jacobian(pt)
# the value of a (sub)gradient of g is available only if the output shape of g is (1,1)
g.subgradient(pt)
# otherwise it will result in an error
f.subgradient(pt) # -> Error
# the value of the hessian of g computed at pt is
g.hessian(pt)
```

For affine and quadratic functions, a method called `get_parameters` is implemented, which returns the matrices and vectors that define those functions. The generic form for an affine function is $A^\top x + b$ while the one for a quadratic form is $x^\top P x + q^\top x + r$:

```
f = A @ x + b
f.get_parameters() # -> A, b
```

3.2.2 Defining constraints from functions

Constraints are represented in the canonical forms $f(x) = 0$ and $f(x) \leq 0$.

They are directly obtained from functions:

```
constraint = g == 0 # g(x) = 0
constraint = g >= 0 # g(x) >= 0
constraint = g <= 0 # g(x) <= 0
```

On the right side of (in)equalities, numpy arrays and functions (with appropriate shapes) are also allowed:

```
c = np.random.rand(2,1)
constr = f <= c
```

which is automatically translated in the corresponding canonical form.

Constraints can be evaluated at any point by using the `eval` method which returns a boolean value if the constraint is satisfied. Moreover, the function defining a constraints can be retrieved with the `function` method:

```
pt = np.random.rand(2,1)
constr.eval(pt) # -> True if f(pt) <= c
constr.function.eval(pt) # -> value of f - c at pt
```

Affine and quadratic constraints

Parameters defining affine and quadratic constraints can be easily obtained. They can be accessed by calling the `get_parameters` method:

```
f = A @ x + b
constraint = f == 0 # affine equality constraint
# f has the form A^T x + b
constraint.get_parameters() # returns A and b
```

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```
g = f @ f
constraint = g == 0 # quadratic equality constraint
# g has the form x^T P x + q^T x + r
constraint.get_parameters() # returns P, q and r
```

Projection onto a constraint set

The projection of a point onto the set defined by a constraint can be computed via the `projection` method:

```
projected_point = f.projection(pt)
```

Constraint sets

Some particular constraint sets (for which projection of points is easy to compute) are also available through specialized classes, which are extensions of the class `Constraint`. For instance, suppose you want all the components of x to be in $[-1, 1]$. Then you can define a `Box` constraint as:

```
from disropt.constraints import Box
bound = np.ones((2,1))
constr = Box(-bound, bound)
```

Two methods are available: `projection` and `intersection`. The first one returns the projection of a given point on the set, while the second one intersects the set with another one. This feature is particularly useful in set-membership estimation algorithms.

Constraint sets can be converted into a list of constraints through the method `to_constraints`.

3.3 Local data of optimization problems

The class `Problem` allows one to define and solve optimization problems of various types. It is discussed in detail in *a dedicated section*.

In the distributed framework of **disropt**, the `Problem` class is also meant to specify local data (available to the agent) of global optimization problems. The class should be used in different ways, depending on the distributed optimization set-up (refer to the *general forms*), and must be provided to the agent (see also *Quick start*).

3.3.1 Cost-coupled set-up

For the cost-coupled set-up, the two objects that can be specified are

- the local contribution to the cost function, i.e., the function $f_i(x)$
- the local constraints (if any), i.e., the set X_i (which must be described through a list of constraints).

To this end, create an instance of the class `Problem` and set the objective function to $f_i(x)$ and the constraints to X_i.

For instance, suppose $x \in \mathbb{R}^2$ and assume the agent knows

$$f_i(x) = \|x\|^2, \qquad X_i = \{x \mid -1 \leq x \leq 1\}.$$

The corresponding Python code is:

```
from disropt.functions import SquaredNorm, Variable
from disropt.problems import Problem
x = Variable(2)
objective_function = SquaredNorm(x)
constraints = [x >= -1, x <= 1]
problem = Problem(objective_function, constraints)
```

If there are no local constraints (in the example $X_i \equiv \mathbb{R}^2$), then no constraints should be passed to `Problem`:

```
x = Variable(2)
objective_function = SquaredNorm(x)
problem = Problem(objective_function) # no constraints
```

3.3.2 Common-cost set-up

For the common-cost set-up, the objective function $f(x)$ is assumed to be known by all the agents. Theerefore, the two objects that must be specified are

- the global cost function, i.e., the function $f(x)$
- the local constraints, i.e., the set X_i (which must be described through a list of constraints).

To this end, create an instance of the class `Problem` and set the objective function to $f(x)$ and the constraints to X_i.

For instance, suppose $x \in \mathbb{R}^2$ and assume the agent knows

$$f(x) = \|x\|^2, \qquad X_i = \{x \mid -1 \leq x \leq 1\}.$$

The corresponding Python code is:

```
from disropt.functions import SquaredNorm, Variable
from disropt.problems import Problem
x = Variable(2)
objective_function = SquaredNorm(x)
constraints = [x >= -1, x <= 1]
problem = Problem(objective_function, constraints)
```

3.3.3 Constraint-coupled set-up

For the constraint-coupled set-up, the three objects that can be specified are

- the local contribution to the cost function, i.e., the function $f_i(x_i)$
- the local contribution to the coupling constraints, i.e., the function $g_i(x_i)$
- the local constraints (if any), i.e., the set X_i (which must be described through a list of constraints).

To this end, create an instance of the class `ConstraintCoupledProblem` and set the objective function to $f_i(x_i)$, the coupling function to $g_i(x_i)$ and the constraints to X_i.

For instance, suppose $x_i \in \mathbb{R}^2$ and assume the agent knows

$$f_i(x_i) = \|x_i\|^2, \qquad g_i(x_i) = x_i, \qquad X_i = \{x \mid -1 \leq x \leq 1\}.$$

The corresponding Python code is:

```
from disropt.functions import SquaredNorm, Variable
from disropt.problems import ConstraintCoupledProblem
x = Variable(2)
objective_function = SquaredNorm(x)
coupling_function = x
constraints = [x >= -1, x <= 1]
problem = ConstraintCoupledProblem(objective_function, constraints, coupling_function)
```

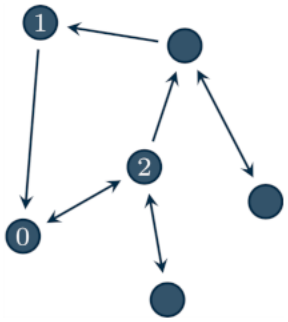
If there are no local constraints (in the example $X_i \equiv \mathbb{R}^2$), then no constraints should be passed to `ConstraintCoupledProblem`:

```
x = Variable(2)
objective_function = SquaredNorm(x)
coupling_function = x
problem = ConstraintCoupledProblem(
    objective_function=objective_function,
    coupling_function=coupling_function) # no local constraints
```

3.4 Agents in the network

The `Agent` class is meant to represent the local computing units that collaborate in the network in order to solve some specific problem.

Agents are instantiated by defining their in/out-neighbors and the weights they assign their neighbors. For example, consider the following network



Then, agent 0 is defined as:

```
from disropt.agents import Agent

agent = Agent(in_neighbors=[1,2],
              out_neighbors=[2],
              weights=[0.3, 0.2])
```

3.4.1 Local data of an optimization problem

Assigning a local optimization problem to an agent is done via the `set_problem` method, which modifies the `problem` attribute of the agent.

Assume that the variable `problem` contains the local problem data, according to the procedure described in the *previous page*. Then, the variable is assigned to the agent by:

```
agent.set_problems(problem)
```

Local objective functions, constraints and all the operations related to the problem can be accessed through the attribute `problem`. For example:

```
agent.problem.objective_function.eval(pt) # evalate the objective function at pt
agent.problem.constraints # -> return the list of local constraints
```

3.5 Algorithms

In **disropt**, there are many implemented distributed optimization algorithms. Each algorithm is tailored for a specific distributed optimization set-up (see *Tutorial*).

3.5.1 Basic

- *Consensus* (standard and block wise, synchronous and asynchronous)

3.5.2 Cost-coupled set-up

- *Distributed Subgradient* (standard and block wise)
- *Gradient Tracking*
- *Distributed Dual Decomposition*
- *Distributed ADMM*
- *ASYMM*

3.5.3 Common cost set-up

- *Constraints Consensus*
- *Set membership* (synchronous and asynchronous)

3.5.4 Constraint-coupled set-up

- *Distributed Dual Subgradient*
- *Distributed Primal Decomposition*

3.5.5 Miscellaneous

- *Logic AND* (synchronous and asynchronous)

CHAPTER

FOUR

EXAMPLES

Here we report some examples that show how to use **disropt**. We divide the examples into two groups:

- *Basic examples*: to show how each algorithm should be used
- *Complex examples*: realistic application scenarios, e.g., to make comparative studies on different algorithms

4.1 Basic examples

We provide an example for each implemented distributed algorithm (see also the *list of implemented algorithms*).

4.1.1 Consensus algorithms

Classical consensus

The classical consensus algorithm is implemented through the `Consensus` class.

From the perspective of agent i the classical consensus algorithm works as follows. For $k = 0, 1, \dots$

$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_j^k$$

where $x_i \in \mathbb{R}^n$ and w_{ij} is the weight assigned by agent i to agent j. Usually, average consensus (i.e., the convergence of the local estimate sequences to the initial average) is guaranteed only if the weights w_{ij} form a doubly-stochastic matrix. Otherwise, agreement is still reached but at some other point.

In order to simulate a consensus algorithm over a static undirected graph (with a doubly-stochastic weight matrix), create a file containing the following code and call it *launcher.py*

Listing 1: launcher.py

```
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import Consensus
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()
```

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```
# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
W = metropolis_hastings(Adj)

# reset local seed
np.random.seed()

# create local agent
agent = Agent(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
              out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
              in_weights=W[local_rank, :].tolist())

# instantiate the consensus algorithm
n = 4  # decision variable dimension (n, 1)
x0 = np.random.rand(n, 1)
algorithm = Consensus(agent=agent,
                      initial_condition=x0,
                      enable_log=True)  # enable storing of the generated sequences

# run the algorithm
sequence = algorithm.run(iterations=100)

# print solution
print("Agent {}: {}".format(agent.id, algorithm.get_result()))

# save data
np.save("agents.npy", nproc)
np.save("agent_{}_sequence.npy".format(agent.id), sequence)
```

And then execute it with the desired number of agents:

```
mpirun -np 12 --oversubscribe python launcher.py
```

where the flag `--oversubscribe` is necessary only if the requested number of agents (12 in this case) is higher than the available number of cores (or computing units).

Plot the generated sequences

In order to plot the local sequences generated by the algorithm, we create the file *results.py*.

Listing 2: results.py

```
import numpy as np
import matplotlib.pyplot as plt

# Load the number of agents
N = np.load("agents.npy")

# Load the locally generated sequences
sequence = {}
for i in range(N):
    filename = "agent_{}_sequence.npy".format(i)
```

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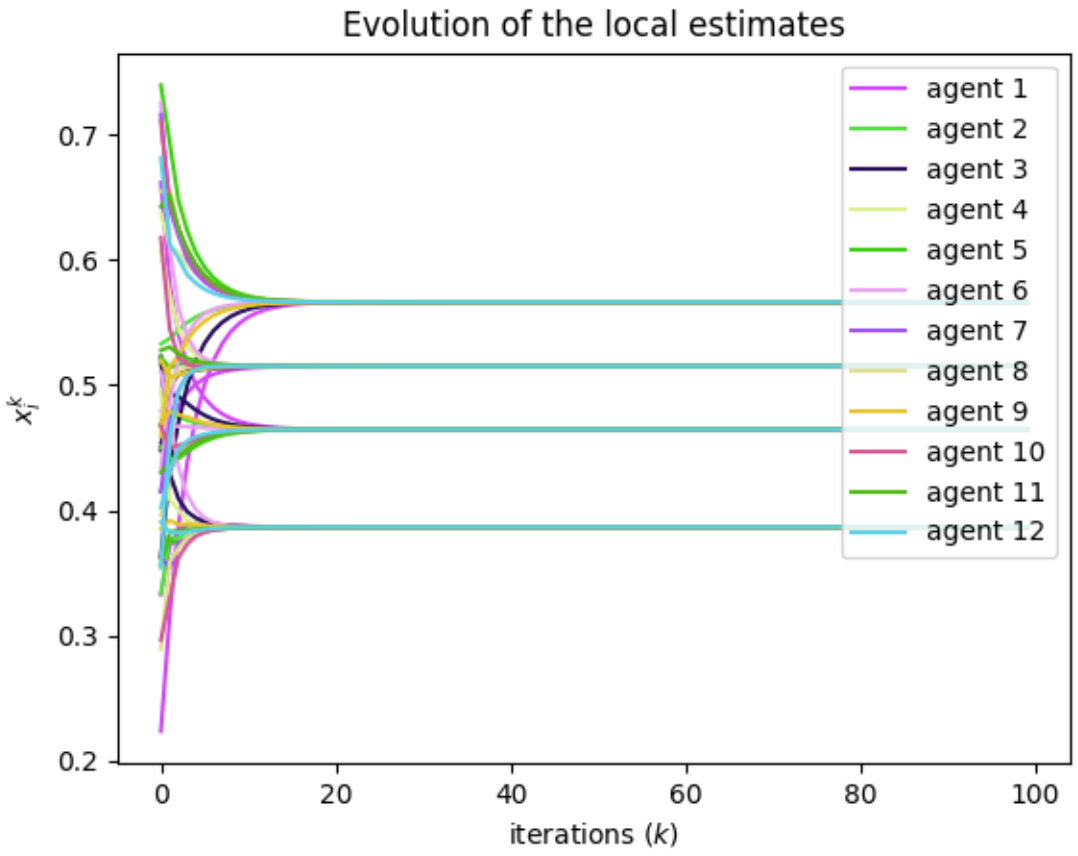
```
    sequence[i] = np.load(filename)

# Plot the evolution of the local estimates
# generate N colors
colors = {}
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()
# generate figure
plt.figure()
for i in range(N):
    dims = sequence[i].shape
    iterations = dims[0]
    for j in range(dims[1]):
        if j == 0: # to avoid generating multiple labels
            plt.plot(np.arange(iterations), sequence[i][:, j, 0], color=colors[i], label=
↪'agent {}'.format(i+1))
        else:
            plt.plot(np.arange(iterations), sequence[i][:, j, 0], color=colors[i])
plt.legend(loc='upper right')
plt.title("Evolution of the local estimates")
plt.xlabel(r"iterations ($k$)")
plt.ylabel(r"$x_i^k$")
plt.savefig('results_fig.png')
plt.show()
```

We execute *results.py* through:

```
python results.py
```



Time-varying graphs

Convergence of consensus algorithms is still achieved over time-varying (possibly directed) graphs, provided that they are jointly strongly connected.

In this case, one can use the `set_neighbors` and `set_weights` methods of the `Agent` class in order to modify the communication network when necessary. This can be done in various ways, for example by overriding the `run` method of the algorithm of by calling it multiple times over different graphs. For example, suppose that the graph changes every 100 iterations and that you want to perform 1000 iterations, then:

```
for g in range(10):
    # generate a new common graph (everyone use the same seed)
    Adj = construct_graph(nproc, p=0.3, seed=g)
    W = metropolis_hastings(Adj)

    # set new neighbors and weights
    agent.set_neighbors(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
                        out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())
    agent.set_weights(weights=W[local_rank, :].tolist())

    algorithm.run(iterations=100)
```

Block-wise consensus

Consensus can be also performed block-wise with respect to the decision variable by using the `BlockConsensus` class.

From the perspective of agent i the algorithm works as follows. At iteration k, if the agent is awake, it selects a random block ℓ_i^k of its local solution and updates

$$x_{i,\ell}^{k+1} = \begin{cases} \sum_{j\in\mathcal{N}_i} w_{ij} x_{j|i,\ell}^k & \text{if} \ell = \ell_i^k \\ x_{i,\ell}^k & \text{otherwise} \end{cases}$$

where $\mathcal{N}_i$ is the current set of in-neighbors and $x_{j|i}, j \in \mathcal{N}_i$ is the local copy of x_j available at node i and $x_{i,\ell}$ denotes the ℓ-th block of x_i. Otherwise $x_i^{k+1} = x_i^k$.

Moreover, at each iteration, each agent can update its local estimate or not at each iteration according to a certain probability (awakening_probability), thus modeling some *asyncrhony*.

The algorithm can be istantiated by providing a list of blocks of the decision variable and the probabilities of drawing each block:

```
algorithm = BlockConsensus(agent=agent,
                           initial_condition=x0,
                           enable_log=True,
                           blocks_list=[(0, 1), (2, 3)],
                           probabilities=[0.3, 0.7],
                           awakening_probability=0.5)
```

Asynchronous consensus

Asynchronous consensus can be seen as a sort of synchronous consensus over time-varying graphs with delays (which may model non negligible computation times and unreliable links) and it is implemented in the `AsynchronousConsensus` class.

When running this algorithm, you can control the computation and sleep times of each agent and the communication channels failure probabilities. Moreover, when running asynchronous algorithms, you have to set the total duration of the execution (and not the number of iterations). We provide the following example.

```
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import AsynchronousConsensus
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings


# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
W = metropolis_hastings(Adj)

# reset local seed
np.random.seed()
```

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```
# create local agent
agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
    in_weights=W[local_rank, :].tolist())

# instantiate the asynchronous consensus algorithm
x0 = np.random.rand(2, 1)
algorithm = AsynchronousConsensus(agent=agent,
                                  initial_condition=x0,
                                  enable_log=True,
                                  force_sleep=True,
                                  maximum_sleep=0.1,
                                  sleep_type="random",
                                  force_computation_time=True,
                                  maximum_computation_time=0.1,
                                  computation_time_type="random",
                                  force_unreliable_links=True,
                                  link_failure_probability=0.1)

# run the algorithm
timestamp_sequence_awake, timestamp_sequence_sleep, sequence = algorithm.run(running_
↪time=4)

# print solution
print("Agent {}: {}".format(agent.id, algorithm.get_result()))

# save data
np.save("agents.npy", nproc)
np.save("agent_{}_sequence.npy".format(agent.id), sequence)
np.save("agent_{}_timestamp_sequence_awake.npy".format(agent.id), timestamp_sequence_
↪awake)
np.save("agent_{}_timestamp_sequence_sleep.npy".format(agent.id), timestamp_sequence_
↪sleep)
```

Plot the generated sequences

```
import numpy as np
import matplotlib.pyplot as plt


# number of agents
N = np.load("agents.npy")

# retrieve local sequences
sequence = {}
timestamp_sequence_awake = {}
timestamp_sequence_sleep = {}
colors = {}
t_init = None
for i in range(N):
```

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```
    colors[i] = np.random.rand(3, 1).flatten()
    filename = "agent_{}_sequence.npy".format(i)
    sequence[i] = np.load(filename, allow_pickle=True)

    filename = "agent_{}_timestamp_sequence_awake.npy".format(i)
    timestamp_sequence_awake[i] = np.load(filename, allow_pickle=True)

    filename = "agent_{}_timestamp_sequence_sleep.npy".format(i)
    timestamp_sequence_sleep[i] = np.load(filename, allow_pickle=True)

    if t_init is not None:
        m = min(timestamp_sequence_awake[i])
        t_init = min(t_init, m)
    else:
        t_init = min(timestamp_sequence_awake[i])

for i in range(N):
    timestamp_sequence_awake[i] = timestamp_sequence_awake[i] - t_init
    timestamp_sequence_sleep[i] = timestamp_sequence_sleep[i] - t_init

# plot
plt.figure()
colors = {}
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()

    dims = sequence[i].shape
    for j in range(dims[1]):
        if j == 0:
            plt.plot(timestamp_sequence_sleep[i], sequence[i][:, j, 0],
                     color=colors[i],
                     label="Agent {}: awakenings={}".format(i+1, dims[0]))
        else:
            plt.plot(timestamp_sequence_sleep[i], sequence[i][:, j, 0],
                     color=colors[i])
plt.xlabel("time (s)")
plt.ylabel("x_i")
plt.title("Local estimates sequences")
plt.legend()

plt.savefig('sequences.png')

S = {}
for i in range(N):
    aw = np.array(timestamp_sequence_awake[i])
    aw = np.vstack([aw, np.zeros(aw.shape)])
    sl = np.array(timestamp_sequence_sleep[i])
    sl = np.vstack([sl, np.ones(sl.shape)])
    aux = np.hstack([aw, sl]).transpose()
    signal = aux[aux[:, 0].argsort()]
    inverse_signal = np.zeros(signal.shape)
    inverse_signal += signal
```

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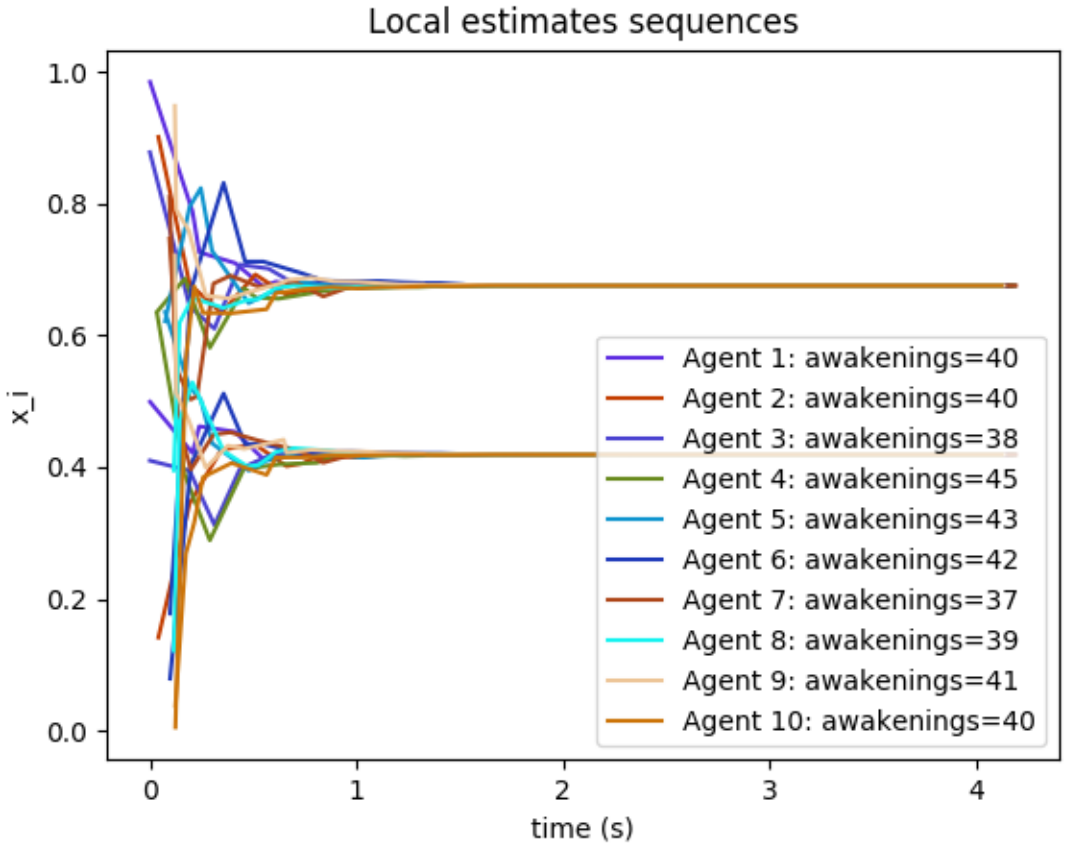
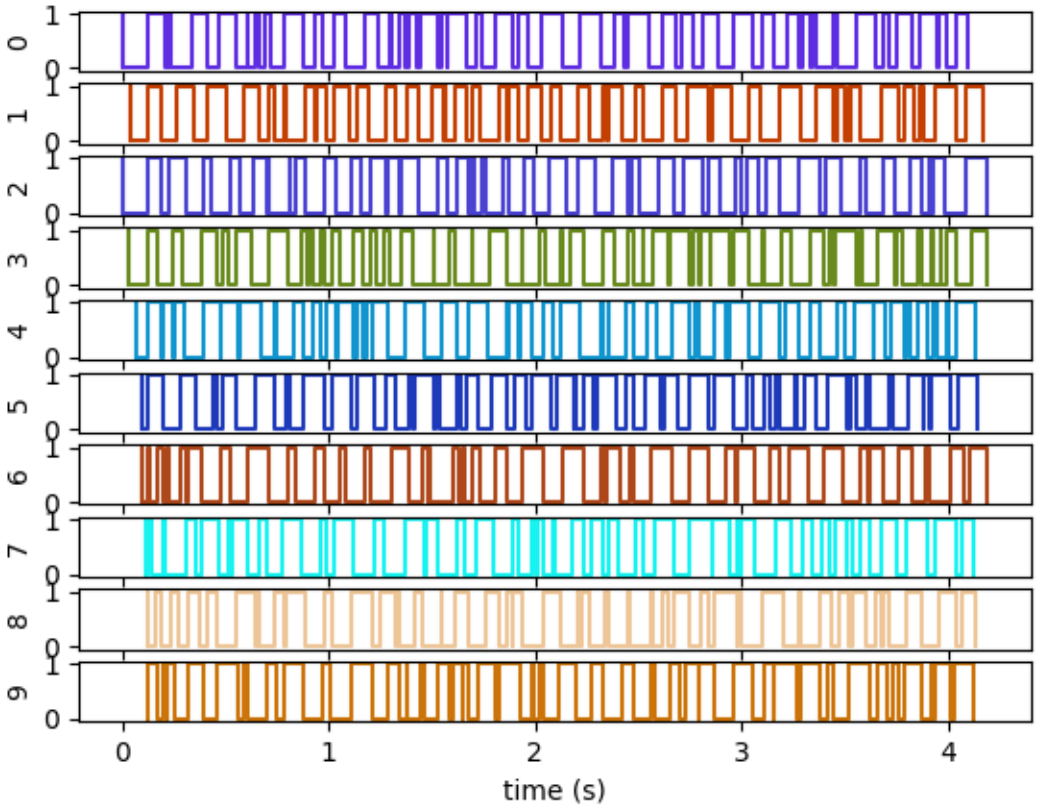
```
    inverse_signal[:, 1] = abs(inverse_signal[:, 1] - 1)
    ww = np.empty([signal.shape[0]*2, 2])
    ww[::2] = signal
    ww[1::2] = inverse_signal
    S[i] = ww

fig, axes = plt.subplots(int(N),1, sharex=True)
for i in range(N):
    axes[i].plot(S[i][:, 0], S[i][:, 1], color=colors[i])
    axes[i].set_ylabel("{}".format(i))

plt.xlabel("time (s)")

plt.savefig('sleep_awake.png')

plt.show()
```





4.1.2 Distributed Logic AND

This is an example on how to use the `LogicAnd` class.

Listing 3: examples/setups/logic-and/launcher.py

```
import numpy as np
import networkx as nx
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.misc import LogicAnd
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.1, seed=1)
W = metropolis_hastings(Adj)
graph = nx.DiGraph(Adj)
graph_diameter = nx.diameter(graph)
```

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```
# create local agent
agent = Agent(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
              out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
              in_weights=W[local_rank, :].tolist())

# instantiate the logic-and algorithm
flag = True
algorithm = LogicAnd(agent, graph_diameter, flag=flag)

algorithm.run(maximum_iterations=100)

print(algorithm.S)
```

The file can be executed by issuing the following command in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
```

4.1.3 Distributed Max Consensus

This is an example on how to use the `MaxConsensus` class.

Listing 4: examples/setups/max-consensus/launcher.py

```
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.misc import MaxConsensus
from disropt.utils.graph_constructor import ring_graph

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = ring_graph(nproc)
graph_diam = nproc-1
n_vars = 3

# create local agent
agent = Agent(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
              out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())

# instantiate the max-consensus algorithm
np.random.seed(local_rank*5)

x0 = np.random.rand(n_vars)*10
algorithm = MaxConsensus(agent, x0, graph_diam, True)

print('Initial value of agent {}:\n {}'.format(local_rank, x0))
```

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```
# execute algorithm
x_sequence = algorithm.run(iterations=100)

# get result
x_final = algorithm.get_result()

print('Result of agent {}:\n {}'.format(local_rank, x_final))
```

The file can be executed by issuing the following command in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
```

4.1.4 Distributed Set Membership

This is an example on how to use the `SetMembership` class. See also the reference [FaGa18].

Listing 5: examples/setups/set_membership/launcher.py

```
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import SetMembership
from disropt.constraints.projection_sets import CircularSector
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()


# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
W = metropolis_hastings(Adj)

# reset local seed
np.random.seed(10*local_rank)


# create local agent
agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
    in_weights=W[local_rank, :].tolist())


# dimension of the variable
n = 2
```

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```
def rotate(p, c, theta):  # rotate p around c by theta (rad)
    xr = np.cos(theta)*(p[0]-c[0])-np.sin(theta)*(p[1]-c[1]) + c[0]
    yr = np.sin(theta)*(p[0]-c[0])+np.cos(theta)*(p[1]-c[1]) + c[1]
    return np.array([xr, yr]).reshape(p.shape)


def measure_generator():  # define a measure generator
    np.random.seed(1)
    common_point = np.random.randn(n, 1)
    np.random.seed(10*local_rank)

    position = np.random.randn(n, 1)
    eps_ang = 2*1e-1
    eps_dist = 1e-1

    np.random.seed()
    rd = np.linalg.norm(common_point-position, 2)
    rd2 = rd + eps_dist * np.random.rand()
    measure = position + rd2*(common_point-position)/rd
    rotation = eps_ang * (np.random.rand()-0.5)
    measure = rotate(measure, position, rotation)

    vect = measure - position
    angle = np.arctan2(vect[1], vect[0])
    radius = np.linalg.norm(vect) + eps_dist*np.random.rand()
    return CircularSector(vertex=position,
                          angle=angle,
                          radius=radius,
                          width=2*eps_ang)




# Run algorithm
algorithm = SetMembership(agent, np.random.rand(n, 1), enable_log=True)
# assign measure generator to the agent
algorithm.set_measure_generator(measure_generator)
sequence = algorithm.run(iterations=1000)

# print solution
print("Agent {}: {}".format(agent.id, algorithm.get_result()))

# save data
np.save("agents.npy", nproc)
np.save("agent_{}_sequence.npy".format(agent.id), sequence)
```

Listing 6: examples/setups/set_membership/results.py

```
import numpy as np
import matplotlib.pyplot as plt

N = np.load("agents.npy")
```

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```
sequence = {}
for i in range(N):
    filename = "agent_{}_sequence.npy".format(i)
    sequence[i] = np.load(filename)

plt.figure()
for i in range(N):
    dims = sequence[i].shape
    iterations = dims[0]
    for j in range(dims[1]):
        plt.plot(np.arange(iterations), sequence[i][:, j, 0])
plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.5 Distributed Subgradient

This is an example on how to use the `SubgradientMethod` class. See also the reference [NeOz09].

Listing 7: examples/setups/distributed_subgradient/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.subgradient import SubgradientMethod
from disropt.functions import QuadraticForm, Variable
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings
from disropt.problems import Problem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
W = metropolis_hastings(Adj)

# reset local seed
np.random.seed(10*local_rank)

agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
    in_weights=W[local_rank, :].tolist())
```

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```
# variable dimension
n = 2

# declare a variable
x = Variable(n)

# define the local objective function
P = np.random.rand(n, n)
P = P.transpose() @ P
bias = np.random.rand(n, 1)
fn = QuadraticForm(x - bias, P)


# define a (common) constraint set
constr = [x<= 1, x >= -1]

# local problem
pb = Problem(fn, constr)
agent.set_problem(pb)

# instantiate the algorithms
initial_condition = 10*np.random.rand(n, 1)

algorithm = SubgradientMethod(agent=agent,
                              initial_condition=initial_condition,
                              enable_log=True)


def step_gen(k): # define a stepsize generator
    return 1/((k+1)**0.51)


# run the algorithm
sequence = algorithm.run(iterations=1000, stepsize=step_gen)
print("Agent {}: {}".format(agent.id, algorithm.get_result().flatten()))

np.save("agents.npy", nproc)

# save agent and sequence
np.save("agent_{}_sequence.npy".format(agent.id), sequence)
with open('agent_{}_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(fn, output, pickle.HIGHEST_PROTOCOL)
```

Listing 8: examples/setups/distributed_subgradient/results.py

```
import numpy as np
import matplotlib.pyplot as plt
import pickle

N = np.load("agents.npy")
n = 2
```

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```
sequence = {}
local_function = {}
for i in range(N):
    filename = "agent_{}_sequence.npy".format(i)
    sequence[i] = np.load(filename)
    with open('agent_{}_function.pkl'.format(i), 'rb') as inp:
        local_function[i] = pickle.load(inp)

plt.figure()
colors = {}
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()
    dims = sequence[i].shape
    print(dims)
    iterations = dims[0]
    for j in range(dims[1]):
        plt.plot(np.arange(iterations), sequence[i][:, j, 0], color=colors[i])

function = np.zeros([iterations, 1])
for k in range(iterations):
    avg = np.zeros([n, 1])
    for i in range(N):
        avg += sequence[i][k, :, 0].reshape(n, 1)
    avg = avg/N
    for i in range(N):
        function[k] += local_function[i].eval(avg).flatten()

plt.figure()
plt.semilogy(function)

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.6 Gradient Tracking

This is an example on how to use the `GradientTracking` class.

Listing 9: examples/setups/gradient_tracking/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.gradient_tracking import GradientTracking
from disropt.functions import QuadraticForm, Variable
from disropt.utils.utilities import is_pos_def
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings
```

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```
from disropt.problems.problem import Problem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
W = metropolis_hastings(Adj)

# reset local seed
np.random.seed()

agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
    in_weights=W[local_rank, :].tolist())

# variable dimension
d = 4

# generate a positive definite matrix
P = np.random.randn(d, d)
while not is_pos_def(P):
    P = np.random.randn(d, d)
bias = np.random.randn(d, 1)
# declare a variable
x = Variable(d)

# define the local objective function
fun = QuadraticForm(x - bias, P)

# local problem
pb = Problem(fun)
agent.set_problem(pb)

# instantiate the algorithms
initial_condition = np.random.rand(d, 1)

algorithm = GradientTracking(agent=agent,
                             initial_condition=initial_condition,
                             enable_log=True)



# run the algorithm
sequence = algorithm.run(iterations=1000, stepsize=0.001)
print("Agent {}: {}".format(agent.id, algorithm.get_result().flatten()))

np.save("agents.npy", nproc)
```

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```
# save agent and sequence
with open('agent_{}_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.objective_function, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_sequence.npy".format(agent.id), sequence)
```

Listing 10: examples/setups/gradient_tracking/results.py

```
import numpy as np
import matplotlib.pyplot as plt
import pickle

N = np.load("agents.npy")
d = 4

sequence = {}
local_function = {}
for i in range(N):
    filename = "agent_{}_sequence.npy".format(i)
    sequence[i] = np.load(filename)
    with open('agent_{}_function.pkl'.format(i), 'rb') as input:
        local_function[i] = pickle.load(input)

plt.figure()
colors = {}
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()
    dims = sequence[i].shape
    print(dims)
    iterations = dims[0]
    for j in range(dims[1]):
        plt.plot(np.arange(iterations), sequence[i][:, j, 0], color=colors[i])

function = np.zeros([iterations, 1])
for k in range(iterations):
    avg = np.zeros([d, 1])
    for i in range(N):
        avg += sequence[i][k, :, 0].reshape(d, 1)
    avg = avg/N
    for i in range(N):
        function[k] += local_function[i].eval(avg).flatten()

plt.figure()
plt.plot(function)

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.7 Distributed Dual Decomposition

This is an example on how to use the `DualDecomposition` class.

Listing 11: examples/setups/distributed_dual_decomposition/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.dual_decomp import DualDecomposition
from disropt.functions import QuadraticForm, Variable
from disropt.utils.graph_constructor import binomial_random_graph
from disropt.problems import Problem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)

# reset local seed
np.random.seed(10*local_rank)

agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())

# variable dimension
n = 2

# generate a positive definite matrix
P = np.random.randn(n, n)
P = P.transpose() @ P
bias = 3*np.random.randn(n, 1)
# declare a variable
x = Variable(n)

# define the local objective function
fn = QuadraticForm(x - bias, P)

# define a (common) constraint set
constr = [np.eye(n) @ x <= 1, np.eye(n) @ x >= -1]

# local problem
pb = Problem(fn, constr)
agent.set_problem(pb)

# instantiate the algorithms
initial_condition = {}
```

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```
for j in agent.in_neighbors:
    initial_condition[j] = 10*np.random.rand(n, 1)

algorithm = DualDecomposition(agent=agent,
                              initial_condition=initial_condition,
                              enable_log=True)


def step_gen(k): # define a stepsize generator
    return 1/((k+1)**0.51)


# run the algorithm
x_sequence, lambda_sequence = algorithm.run(iterations=100, stepsize=step_gen)
x_t, lambda_t = algorithm.get_result()
print("Agent {}: primal {} dual {}".format(agent.id, x_t.flatten(), lambda_t))

np.save("agents.npy", nproc)

# save agent and sequence
with open('agent_{}_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.objective_function, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_dual_sequence.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(lambda_sequence, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_primal_sequence.npy".format(agent.id), x_sequence)
```

Listing 12: examples/setups/distributed_dual_decomposition/results.py

```
import dill as pickle
import numpy as np
import matplotlib.pyplot as plt

N = np.load("agents.npy")
n = 2

lambda_sequence = {}
x_sequence = {}
local_obj_function = {}
for i in range(N):
    with open('agent_{}_dual_sequence.pkl'.format(i), 'rb') as input:
        lambda_sequence[i] = pickle.load(input)
    x_sequence[i] = np.load("agent_{}_primal_sequence.npy".format(i))
    with open('agent_{}_function.pkl'.format(i), 'rb') as input:
        local_obj_function[i] = pickle.load(input)

# plot sequence of x
plt.figure()
plt.title("Primal sequences")

colors = {}
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()
```

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```
    dims = x_sequence[i].shape
    print(dims)
    iterations = dims[0]
    for j in range(dims[1]):
        plt.plot(np.arange(iterations), x_sequence[i][:, j, 0], color=colors[i])

# plot primal cost
plt.figure()
plt.title("Primal cost")

function = np.zeros([iterations, 1])
for k in range(iterations):
    avg = np.zeros([n, 1])
    for i in range(N):
        avg += x_sequence[i][k, :, 0].reshape(n, 1)
    avg = avg/N
    for i in range(N):
        function[k] += local_obj_function[i].eval(avg).flatten()

plt.plot(np.arange(iterations), function)

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.8 Distributed ADMM

This is an example on how to use the `ADMM` class.

Listing 13: examples/setups/distributed_ADMM/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.admm import ADMM
from disropt.functions import QuadraticForm, Variable
from disropt.utils.graph_constructor import binomial_random_graph
from disropt.problems import Problem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
```

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```
# reset local seed
np.random.seed(10*local_rank)

agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())

# variable dimension
n = 2

# generate a positive definite matrix
P = np.random.randn(n, n)
P = P.transpose() @ P
bias = 3*np.random.randn(n, 1)

# declare a variable
x = Variable(n)

# define the local objective function
fn = QuadraticForm(x - bias, P)

# define a (common) constraint set
constr = [x <= 1, x >= -1]

# local problem
pb = Problem(fn, constr)
agent.set_problem(pb)

# instantiate the algorithms
initial_z = np.ones((n, 1))
# initial_lambda = {local_rank: 10*np.random.rand(n, 1)}
initial_lambda = {local_rank: np.ones((n, 1))}

for j in agent.in_neighbors:
    # initial_lambda[j] = 10*np.random.rand(n, 1)
    initial_lambda[j] = np.ones((n, 1))

algorithm = ADMM(agent=agent,
                 initial_lambda=initial_lambda,
                 initial_z=initial_z,
                 enable_log=True)

# run the algorithm
x_sequence, lambda_sequence, z_sequence = algorithm.run(iterations=100, penalty=0.1,␣
↪verbose=True)
x_t, lambda_t, z_t = algorithm.get_result()
print("Agent {}: primal {} dual {} auxiliary primal {}".format(agent.id, x_t.flatten(),␣
↪lambda_t, z_t.flatten()))

np.save("agents.npy", nproc)

# save agent and sequence
```

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```
if local_rank == 0:
    with open('constraints.pkl', 'wb') as output:
        pickle.dump(constr, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.objective_function, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_dual_sequence.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(lambda_sequence, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_primal_sequence.npy".format(agent.id), x_sequence)
np.save("agent_{}_auxiliary_primal_sequence.npy".format(agent.id), z_sequence)
```

Listing 14: examples/setups/distributed_ADMM/results.py

```
import dill as pickle
import numpy as np
import matplotlib.pyplot as plt
from disropt.problems import Problem

N = np.load("agents.npy")
n = 2

lambda_sequence = {}
x_sequence = {}
z_sequence = {}
local_obj_function = {}
for i in range(N):
    with open('agent_{}_dual_sequence.pkl'.format(i), 'rb') as input:
        lambda_sequence[i] = pickle.load(input)
    x_sequence[i] = np.load("agent_{}_primal_sequence.npy".format(i))
    z_sequence[i] = np.load("agent_{}_auxiliary_primal_sequence.npy".format(i))
    with open('agent_{}_function.pkl'.format(i), 'rb') as input:
        local_obj_function[i] = pickle.load(input)
with open('constraints.pkl', 'rb') as input:
    constr = pickle.load(input)
iters = x_sequence[0].shape[0]

# solve centralized problem
global_obj_func = 0
for i in range(N):
    global_obj_func += local_obj_function[i]

global_pb = Problem(global_obj_func, constr)
x_centr = global_pb.solve()
cost_centr = global_obj_func.eval(x_centr)
x_centr = x_centr.flatten()

# compute cost errors
cost_err = np.zeros((N, iters)) - cost_centr

for i in range(N):
    for t in range(iters):
        # add i-th cost
        cost_err[i, t] += local_obj_function[i].eval(x_sequence[i][t, :])
```

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```
# plot maximum cost error
plt.figure()
plt.title('Maximum cost error (among agents)')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$\max_{i} \: \left|\sum_{j=1}^N f_j(x_i^k) - f^\star \right|$")
plt.semilogy(np.arange(iters), np.amax(np.abs(cost_err), axis=0))

# plot maximum solution error
sol_err = np.zeros((N, iters))
for i in range(N):
    sol_err[i] = np.linalg.norm(np.squeeze(x_sequence[i]) - x_centr[None, :], axis=1)

plt.figure()
plt.title('Maximum solution error (among agents)')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$\max_{i} \: \|x_i^k - x^\star\|$")
plt.semilogy(np.arange(iters), np.amax(sol_err, axis=0))

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.9 ASYMM

This is an example on how to use the `ASYMM` class. See also the reference [FaGa19b].

Listing 15: examples/setups/asymm/launcher.py

```
import numpy as np
import networkx as nx
import dill as pickle
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.asymm import ASYMM
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings
from disropt.functions import SquaredNorm, Norm, Variable
from disropt.problems import Problem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=1, seed=1)
W = metropolis_hastings(Adj)
graph = nx.DiGraph(Adj)
```

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```
graph_diameter = nx.diameter(graph)

if local_rank == 0:
    print(Adj)

# create local agent
agent = Agent(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
              out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
              in_weights=W[local_rank, :].tolist())

# problem set-up
n = 2

# target point
x_true = np.random.randn(n, 1)

if local_rank == 0:
    print(x_true)

# reset local seed
np.random.seed(10 * local_rank)

# local position
c = np.random.randn(n, 1)

# true distance
distance = np.linalg.norm(x_true-c, ord=2)

# declare a variable
x = Variable(n)

# define the local objective function
objective = (x -c) @ (x - c)

# local constraint
upper_bound = (x-c) @ (x-c) <= (distance**2 + 0.00001*np.random.rand())
lower_bound = (x-c) @ (x-c) >= (distance**2 - 0.00001*np.random.rand())

constraints = [upper_bound, lower_bound]
# define local problem
pb = Problem(objective_function=objective, constraints=constraints)
agent.set_problem(pb)

####
x0 = np.random.randn(n, 1)
algorithm = ASYMM(agent=agent,
                  graph_diameter=graph_diameter,
                  initial_condition=x0,
                  enable_log=True)

timestamp_sequence_awake, timestamp_sequence_sleep, sequence = algorithm.run(running_
↪time=30.0)
```

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```
# print solution
print("Agent {}:\n value: {}\n".format(agent.id, algorithm.get_result()))

# save data
with open('agent_{}_constraints.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.constraints, output, pickle.HIGHEST_PROTOCOL)
np.save("agents.npy", nproc)
np.save("agent_{}_sequence.npy".format(agent.id), sequence)
np.save("agent_{}_timestamp_sequence_awake.npy".format(agent.id), timestamp_sequence_
↪awake)
np.save("agent_{}_timestamp_sequence_sleep.npy".format(agent.id), timestamp_sequence_
↪sleep)
```

Listing 16: examples/setups/asymm/results.py

```
import numpy as np
import dill as pickle
import matplotlib.pyplot as plt


# number of agents
N = np.load("agents.npy")

# retrieve local sequences
sequence = {}
constraints = {}
timestamp_sequence_awake = {}
timestamp_sequence_sleep = {}
colors = {}
t_init = None
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()
    filename = "agent_{}_sequence.npy".format(i)
    sequence[i] = np.load(filename, allow_pickle=True)

    filename = "agent_{}_timestamp_sequence_awake.npy".format(i)
    timestamp_sequence_awake[i] = np.load(filename, allow_pickle=True)

    filename = "agent_{}_timestamp_sequence_sleep.npy".format(i)
    timestamp_sequence_sleep[i] = np.load(filename, allow_pickle=True)

    if t_init is not None:
        m = min(timestamp_sequence_awake[i])
        t_init = min(t_init, m)
    else:
        t_init = min(timestamp_sequence_awake[i])

    with open('agent_{}_constraints.pkl'.format(i), 'rb') as input:
        constraints[i] = pickle.load(input)

for i in range(N):
```

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```
    timestamp_sequence_awake[i] = timestamp_sequence_awake[i] - t_init
    timestamp_sequence_sleep[i] = timestamp_sequence_sleep[i] - t_init

# plot
plt.figure()
for i in range(N):
    dims = sequence[i].shape
    for j in range(dims[1]):
        for m in range(dims[2]):
            plt.plot(timestamp_sequence_sleep[i], sequence[i][:, j, m])
plt.ylim([-5, 5])

# # plt
plt.figure()
for i in range(N):
    dims = sequence[i].shape
    iterations = dims[0]
    print("Agent {}, iterations {}".format(i, iterations))
    feasibility = np.zeros([iterations, 2])
    for k in range(iterations):
        for idx, constr in enumerate(constraints[i]):
            flag = constr.eval((sequence[i][k, :, :]).reshape(2,1))
            if flag == False:
                feasibility[k, idx] += abs(constr.function.eval((sequence[i][k, :, :]).
↪reshape(2,1)).flatten())
    plt.semilogy(timestamp_sequence_sleep[i], feasibility)
# plt.lim([-1, 15])

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.10 Constraints Consensus

This is an example on how to use the `ConstraintsConsensus` class. See also the reference [NoBu11].

Listing 17: examples/setups/constraints_consensus/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import ConstraintsConsensus
from disropt.functions import Variable, QuadraticForm
from disropt.utils.graph_constructor import binomial_random_graph
from disropt.problems import Problem

# get MPI info
```

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```
NN = MPI.COMM_WORLD.Get_size()
agent_id = MPI.COMM_WORLD.Get_rank()

# Generate a common graph (everyone uses the same seed)
Adj = binomial_random_graph(NN, p=0.03, seed=1)

######################
# Problem parameters
######################
np.random.seed(10*agent_id)

# linear objective function
dim = 2
z = Variable(dim)
c = np.ones([dim,1])
obj_func = c @ z

# constraints are circles of the form (z-p)^\top (z-p) <= 1
# equivalently z^\top z - 2(A p)^\top z + p^\top A p <= 1
I = np.eye(dim)
p = np.random.rand(dim,1)
r = 1 # unitary radius

constr = []
ff = QuadraticForm(z,I,- 2*(I @ p),(p.transpose() @ I @ p) - r**2)
constr.append(ff<= 0)

######################
# Distributed algorithms
######################

# local agent and problem
agent = Agent(
    in_neighbors=np.nonzero(Adj[agent_id, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, agent_id])[0].tolist())
pb = Problem(obj_func, constr)
agent.set_problem(pb)

# instantiate the algorithm
algorithm = ConstraintsConsensus(agent=agent,
                                 enable_log=True)

# run the algorithm
n_iter = NN*3
x_sequence = algorithm.run(iterations=n_iter, verbose=True)

# print results
x_final = algorithm.get_result()
print("Agent {}: {}".format(agent_id, x_final.flatten()))

# save results to file
if agent_id == 0:
```

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```
    with open('info.pkl', 'wb') as output:
        pickle.dump({'N': NN, 'size': dim, 'iterations': n_iter}, output, pickle.HIGHEST_
↪PROTOCOL)
    with open('objective_function.pkl', 'wb') as output:
        pickle.dump(obj_func, output, pickle.HIGHEST_PROTOCOL)

with open('agent_{}_constr.pkl'.format(agent_id), 'wb') as output:
    pickle.dump(constr, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_seq.npy".format(agent_id), x_sequence)
```

Listing 18: examples/setups/constraints_consensus/results.py

```
import numpy as np
import matplotlib.pyplot as plt
from disropt.problems import Problem
import pickle
import tikzplotlib

# initialize
with open('info.pkl', 'rb') as inp:
    info = pickle.load(inp)
NN = info['N']
iters = info['iterations']
size = info['size']
# load agent data
sequence = np.zeros((NN, iters, size))
local_constr = {}
for i in range(NN):
    sequence[i, :, :] = np.load("agent_{}_seq.npy".format(i), allow_pickle=True).
↪reshape((iters, size))
    with open('agent_{}_constr.pkl'.format(i), 'rb') as inp:
        local_constr[i] = pickle.load(inp)
with open('objective_function.pkl', 'rb') as inp:
    obj_func = pickle.load(inp)

# solve centralized problem
global_constr = []
for i in range(NN):
    global_constr.extend(local_constr[i])
global_pb = Problem(obj_func, global_constr)
x_centr = global_pb.solve()
cost_centr = obj_func.eval(x_centr)

# compute cost errors
cost_err = np.zeros((NN, iters))

for i in range(NN):
    for t in range(iters):
        cost_err[i, t] = abs(obj_func.eval(sequence[i, t, :].reshape((size, 1))) - cost_
↪centr)

# compute max violation
```

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```
vio_err = np.zeros((NN, iters))
for i in range(NN):
    for t in range(iters):
        xt = sequence[i, t, :].reshape((size, 1))
        max_err = np.zeros((len(global_constr), 1))
        for c in range(len(global_constr)):
            max_err[c] = global_constr[c].function.eval(xt)
        vio_err[i, t] = np.max(max_err)

# Plot the evolution of the local estimates
# generate N colors
colors = {}
for i in range(NN):
    colors[i] = np.random.rand(3, 1).flatten()

# plot cost error
plt.figure()
plt.title('Evolution of cost error')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$|f(x_i^k) - f^\star|$")

for i in range(NN):
    plt.plot(np.arange(iters), cost_err[i, :], color=colors[i])

# plot violation error
plt.figure()
plt.title('Maximum constraint violation')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$max_j g(x_i^k)$")

for i in range(NN):
    plt.plot(np.arange(iters), vio_err[i, :], color=colors[i])

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.11 Distributed Simplex

This is an example on how to use the `DistributedSimplex` class. See also the reference [BuNo11].

Listing 19: examples/setups/distributed_simplex/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.constraintexchange import DistributedSimplex
```

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```
from disropt.functions import Variable
from disropt.utils.graph_constructor import ring_graph
from disropt.utils.LP_utils import generate_LP
from disropt.problems import LinearProblem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a ring graph (for which the diameter is nproc-1)
Adj = ring_graph(nproc)
graph_diam = nproc-1

# reset local seed
np.random.seed(1)

# number of constraints
n_constr = 3

# number of columns for each processor
k = 2

# generate a feasible optimization problem of size k * nproc
c_glob, A_glob, b_glob, x_glob = generate_LP(k * nproc, n_constr, 50, constr_form='eq')

# extract the columns assigned to this agent
local_indices = list(np.arange(k*local_rank, k*(local_rank+1)))

c_loc = c_glob[local_indices, :]
A_loc = A_glob[:, local_indices]
b_loc = b_glob

# define the local problem data
x = Variable(k)
obj = c_loc @ x
constr = A_loc.transpose() @ x == b_loc
problem = LinearProblem(objective_function=obj, constraints=constr)

# create agent
agent = Agent(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
        out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())
agent.problem = problem

# instantiate the algorithm
algorithm = DistributedSimplex(agent, enable_log=True, problem_size=nproc*k,
    local_indices=local_indices, stop_iterations=2*graph_diam+1)

# run the algorithm
x_sequence, J_sequence = algorithm.run(iterations=100, verbose=True)

# print results
```

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```
_, _, _, J_final = algorithm.get_result()
print("Agent {} - {} iterations - final cost {}".format(agent.id, len(J_sequence), J_
↪final))

# save results to file
if agent.id == 0:
    with open('info.pkl', 'wb') as output:
        pickle.dump({'N': nproc, 'n_vars': k * nproc, 'n_constr': n_constr, 'c': c_glob,
            'A': A_glob, 'b': b_glob, 'opt_sol': x_glob}, output, pickle.HIGHEST_
↪PROTOCOL)

np.save("agent_{}_x_seq.npy".format(agent.id), x_sequence)
np.save("agent_{}_J_seq.npy".format(agent.id), J_sequence)
```

Listing 20: examples/setups/distributed_simplex/results.py

```
import numpy as np
import matplotlib.pyplot as plt
import dill as pickle
from disropt.functions import Variable
from disropt.problems import LinearProblem

# initialize
with open('info.pkl', 'rb') as inp:
    info = pickle.load(inp)
NN        = info['N']
c         = info['c']
opt_sol   = info['opt_sol']

# load agent data
sequence_J = {}
for i in range(NN):
    sequence_J[i] = np.load("agent_{}_J_seq.npy".format(i))

# compute optimal cost
opt_cost = (c.transpose() @ opt_sol).flatten()

# plot cost evolution
plt.figure()
plt.title("Cost evolution")
colors = np.random.rand(NN+1, 3)
max_iters = 0

for i in range(NN):
    seq_J_i = sequence_J[i]
    n_iters_i = len(seq_J_i)
    max_iters = max(max_iters, n_iters_i)
    plt.plot(np.arange(n_iters_i), seq_J_i, color=colors[i])

# plot optimal cost
plt.plot(np.arange(max_iters), np.ones(max_iters)*opt_cost, '--', color=colors[NN])
plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 10 --oversubscribe python launcher.py
> python results.py
```

4.1.12 Distributed Simplex (dual problem)

This is an example on how to use the `DualDistributedSimplex` class, which forms the dual problem of the given optimization problem and solves it with the Distributed Simplex algorithm.

Listing 21: examples/setups/distributed_simplex/launcher_dual.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.constraintexchange import DualDistributedSimplex
from disropt.functions import Variable
from disropt.utils.graph_constructor import ring_graph
from disropt.utils.LP_utils import generate_LP
from disropt.problems import LinearProblem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()

# Generate a ring graph (for which the diameter is nproc-1)
Adj = ring_graph(nproc)
graph_diam = nproc-1

# reset local seed
np.random.seed(1)

# number of variables
n_vars = 3

# number of constraints for each processor
k = 2

# generate a feasible optimization problem with k * nproc constraints
c_glob, A_glob, b_glob, x_glob = generate_LP(n_vars, k * nproc, 50, direction='max')

# extract the constraints assigned to this agent
local_indices = list(np.arange(k*local_rank, k*(local_rank+1)))

c_loc = c_glob
A_loc = A_glob[local_indices, :]
b_loc = b_glob[local_indices, :]

# define the local problem data
x = Variable(n_vars)
```

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```
obj = -c_loc @ x # minus sign for maximization
constr = A_loc.transpose() @ x <= b_loc
problem = LinearProblem(objective_function=obj, constraints=constr)

# create agent
agent = Agent(in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
        out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())
agent.problem = problem

# instantiate the algorithm
algorithm = DualDistributedSimplex(agent, enable_log=True, num_constraints=nproc*k,
    local_indices=local_indices, stop_iterations=2*graph_diam+1)

# run the algorithm
x_sequence, J_sequence = algorithm.run(iterations=100, verbose=True)

# print results
_, _, _, J_final = algorithm.get_result()
print("Agent {} - {} iterations - final cost {}".format(agent.id, len(J_sequence), J_
↪final))

# save results to file
if agent.id == 0:
    with open('info.pkl', 'wb') as output:
        pickle.dump({'N': nproc, 'n_constr': k * nproc, 'n_vars': n_vars, 'c': c_glob,
            'A': A_glob, 'b': b_glob, 'opt_sol': x_glob}, output, pickle.HIGHEST_
↪PROTOCOL)

np.save("agent_{}_x_seq.npy".format(agent.id), x_sequence)
np.save("agent_{}_J_seq.npy".format(agent.id), J_sequence)
```

Listing 22: examples/setups/distributed_simplex/results_dual.py

```
import numpy as np
import matplotlib.pyplot as plt
import dill as pickle
from disropt.functions import Variable
from disropt.problems import LinearProblem

# initialize
with open('info.pkl', 'rb') as inp:
    info = pickle.load(inp)
NN        = info['N']
c         = info['c']
opt_sol   = info['opt_sol']

# load agent data
sequence_J = {}
for i in range(NN):
    sequence_J[i] = np.load("agent_{}_J_seq.npy".format(i))

# compute optimal cost
```

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```
opt_cost = (c.transpose() @ opt_sol).flatten()

# plot cost evolution
plt.figure()
plt.title("Cost evolution")
colors = np.random.rand(NN+1, 3)
max_iters = 0

for i in range(NN):
    seq_J_i = sequence_J[i]
    n_iters_i = len(seq_J_i)
    max_iters = max(max_iters, n_iters_i)
    plt.plot(np.arange(n_iters_i), seq_J_i, color=colors[i])

# plot optimal cost
plt.plot(np.arange(max_iters), np.ones(max_iters)*opt_cost, '--', color=colors[NN])
plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 10 --oversubscribe python launcher_dual.py
> python results_dual.py
```

4.1.13 Distributed Dual Subgradient

Warning: This example is currently under development

This is an example on how to use the `DualSubgradientMethod` class. See also the reference [FaMa17].

Listing 23: examples/setups/distributed_dual_subgradient/launcher.py

```
# WARNING: this file is currently under development

import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.dual_subgradient import DualSubgradientMethod
from disropt.functions import QuadraticForm, Variable, AffineForm
from disropt.utils.utilities import is_pos_def
from disropt.constraints.projection_sets import Box
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings
from disropt.problems.constraint_coupled_problem import ConstraintCoupledProblem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
local_rank = comm.Get_rank()
```

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```
# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)
W = metropolis_hastings(Adj)

# reset local seed
np.random.seed()

agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist(),
    in_weights=W[local_rank, :].tolist())

# local variable dimension - random in [2,5]
n_i = np.random.randint(2, 6)

# number of coupling constraints
S = 3

# generate a positive definite matrix
P = np.random.randn(n_i, n_i)
while not is_pos_def(P):
    P = np.random.randn(n_i, n_i)
bias = np.random.randn(n_i, 1)

# declare a variable
x = Variable(n_i)

# define the local objective function
fn = QuadraticForm(x - bias, P)

# define the local constraint set
constr = [x>=-2, x<=2]

# define the local contribution to the coupling constraints
A = np.random.randn(S, n_i)
coupling_fn = A.transpose() @ x

# create local problem and assign to agent
pb = ConstraintCoupledProblem(objective_function=fn,
                              constraints=constr,
                              coupling_function=coupling_fn)
agent.set_problem(pb)

# initialize the dual variable
lambda0 = np.random.rand(S, 1)
xhat0   = np.zeros((n_i, 1))

algorithm = DualSubgradientMethod(agent=agent,
                                  initial_condition=lambda0,
                                  initial_runavg=xhat0,
                                  enable_log=True)
```

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```
def step_gen(k): # define a stepsize generator
    return 0.1/np.sqrt(k+1)

# run the algorithm
lambda_sequence, xhat_sequence = algorithm.run(iterations=1000, stepsize=step_gen)
lambda_t, xhat_t = algorithm.get_result()
print("Agent {}: dual {} primal {}".format(agent.id, lambda_t.flatten(), xhat_t.
↪flatten()))

np.save("agents.npy", nproc)

# save agent and sequence
with open('agent_{}_obj_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.objective_function, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_coup_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.coupling_function, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_dual_sequence.npy".format(agent.id), lambda_sequence)
np.save("agent_{}_runavg_sequence.npy".format(agent.id), xhat_sequence)
```

Listing 24: examples/setups/distributed_dual_subgradient/results.py

```
import numpy as np
import matplotlib.pyplot as plt
import pickle

N = np.load("agents.npy")
S = 3

lambda_sequence = {}
xhat_sequence = {}
local_obj_function = {}
local_coup_function = {}
for i in range(N):
    lambda_sequence[i] = np.load("agent_{}_dual_sequence.npy".format(i))
    xhat_sequence[i] = np.load("agent_{}_runavg_sequence.npy".format(i))
    with open('agent_{}_obj_function.pkl'.format(i), 'rb') as input:
        local_obj_function[i] = pickle.load(input)
    with open('agent_{}_coup_function.pkl'.format(i), 'rb') as input:
        local_coup_function[i] = pickle.load(input)

# plot dual solutions
plt.figure()
plt.title("Dual solutions")
colors = {}
for i in range(N):
    colors[i] = np.random.rand(3, 1).flatten()
    dims = lambda_sequence[i].shape
    iterations = dims[0]
    for j in range(dims[1]):
        plt.plot(np.arange(iterations), lambda_sequence[i][:, j, 0], color=colors[i])
```

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```
# plot cost of running average
plt.figure()
plt.title("Primal cost (running average)")

obj_function = np.zeros([iterations, 1])
for k in range(iterations):
    for i in range(N):
        obj_function[k] += local_obj_function[i].eval(xhat_sequence[i][k, :, 0].reshape(-
↪1,1)).flatten()

plt.plot(obj_function)

# plot coupling constraint utilization
plt.figure()
plt.title("Coupling constraint utilization (running average)")

coup_function = np.zeros([iterations, S])
for k in range(iterations):
    for i in range(N):
        coup_function[k] += local_coup_function[i].eval(xhat_sequence[i][k, :, 0].
↪reshape(-1,1)).flatten()

plt.plot(coup_function)

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.1.14 Distributed Primal Decomposition

This is an example on how to use the `PrimalDecomposition` class.

Listing 25: examples/setups/distributed_primal_decomposition/launcher.py

```
import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms.primal_decomp import PrimalDecomposition
from disropt.functions import QuadraticForm, Variable
from disropt.utils.utilities import is_pos_def
from disropt.constraints.projection_sets import Box
from disropt.utils.graph_constructor import binomial_random_graph
from disropt.problems.constraint_coupled_problem import ConstraintCoupledProblem

# get MPI info
comm = MPI.COMM_WORLD
nproc = comm.Get_size()
```

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```
local_rank = comm.Get_rank()

# Generate a common graph (everyone use the same seed)
Adj = binomial_random_graph(nproc, p=0.3, seed=1)

# reset local seed
np.random.seed()

agent = Agent(
    in_neighbors=np.nonzero(Adj[local_rank, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, local_rank])[0].tolist())

# local variable dimension - random in [2,5]
n_i = np.random.randint(2, 6)

# number of coupling constraints
S = 3

# generate a positive definite matrix
P = np.random.randn(n_i, n_i)
P = P.transpose() @ P
bias = np.random.randn(n_i, 1)

# declare a variable
x = Variable(n_i)

# define the local objective function
fn = QuadraticForm(x - bias, P)

# define the local constraint set
low = -2*np.ones((n_i, 1))
up = 2*np.ones((n_i, 1))
constr = Box(low, up)

# define the local contribution to the coupling constraints
A = np.random.randn(S, n_i)
coupling_fn = A.transpose() @ x

# create local problem and assign to agent
pb = ConstraintCoupledProblem(objective_function=fn,
                              constraints=constr,
                              coupling_function=coupling_fn)
agent.set_problem(pb)

# initialize allocation
y0 = np.zeros((S, 1))

algorithm = PrimalDecomposition(agent=agent,
                                initial_condition=y0,
                                enable_log=True)
```

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```
def step_gen(k): # define a stepsize generator
    return 0.1/np.sqrt(k+1)

# run the algorithm
x_sequence, y_sequence, J_sequence = algorithm.run(iterations=1000, stepsize=step_gen,␣
↪M=100.0, verbose=True)
x_t, y_t, J_t = algorithm.get_result()
print("Agent {}: primal {} allocation {}".format(agent.id, x_t.flatten(), y_t.flatten()))

np.save("agents.npy", nproc)

# save agent and sequence
with open('agent_{}_obj_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.objective_function, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_coup_function.pkl'.format(agent.id), 'wb') as output:
    pickle.dump(agent.problem.coupling_function, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_allocation_sequence.npy".format(agent.id), y_sequence)
np.save("agent_{}_primal_sequence.npy".format(agent.id), x_sequence)
```

Listing 26: examples/setups/distributed_primal_decomposition/results.py

```
import numpy as np
import matplotlib.pyplot as plt
import pickle

N = np.load("agents.npy")
S = 3

y_sequence = {}
x_sequence = {}
local_obj_function = {}
local_coup_function = {}
for i in range(N):
    y_sequence[i] = np.load("agent_{}_allocation_sequence.npy".format(i))
    x_sequence[i] = np.load("agent_{}_primal_sequence.npy".format(i))
    with open('agent_{}_obj_function.pkl'.format(i), 'rb') as input:
        local_obj_function[i] = pickle.load(input)
    with open('agent_{}_coup_function.pkl'.format(i), 'rb') as input:
        local_coup_function[i] = pickle.load(input)

# plot cost of primal sequence
plt.figure()
plt.title("Primal cost")

iterations = x_sequence[i].shape[0]
obj_function = np.zeros([iterations, 1])
for k in range(iterations):
    for i in range(N):
        obj_function[k] += local_obj_function[i].eval(x_sequence[i][k, :, 0].reshape(-1,
↪1)).flatten()

plt.semilogy(obj_function)
```

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```
# plot coupling constraint utilization
plt.figure()
plt.title("Coupling constraint utilization")

coup_function = np.zeros([iterations, S])
for k in range(iterations):
    for i in range(N):
        coup_function[k] += local_coup_function[i].eval(x_sequence[i][k, :, 0].reshape(-
↪1,1)).flatten()

plt.plot(coup_function)

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

4.2 Complex examples

We provide three complex examples on realistic applications, one for each optimization set-up (see also the *tutorial introduction*).

4.2.1 Cost-coupled: classification via Logistic Regression

For the *cost-coupled* set-up, we consider a classification scenario [NedOz09]. In this example, a linear model is trained by minimizing the so-called *logistic loss functions*. The complete code of this example is given *at the end of this page*.

Problem formulation

Suppose there are N agents, where each agent i is equipped with m_i points $p_{i,1}, \dots, p_{i,m_i} \in \mathbb{R}^d$ (which represent training samples in a d-dimensional feature space). Moreover, suppose the points are associated to binary labels, that is, each point $p_{i,j}$ is labeled with $\ell_{i,j} \in \{-1, 1\}$, for all $j \in \{1, \dots, m_i\}$ and $i \in \{1, \dots, N\}$. The problem consists of building a linear classification model from the training samples by maximizing the a-posteriori probability of each class. In particular, we look for a separating hyperplane of the form $\{z \in \mathbb{R}^d \mid w^\top z + b = 0\}$, whose parameters w and b can be determined by solving the convex optimization problem

$$\min_{w,b} \sum_{i=1}^{N} \sum_{j=1}^{m_i} \log\left[1 + e^{-(w^\top p_{i,j} + b)\ell_{i,j}}\right] + \frac{C}{2}\|w\|^2,$$

where $C > 0$ is a parameter affecting regularization. Notice that the problem is cost coupled (refer to the *general formulation*), with each local cost function f_i given by

$$f_i(w,b) = \sum_{j=1}^{m_i} \log\left[1 + e^{-(w^\top p_{i,j} + b)\ell_{i,j}}\right] + \frac{C}{2N}\|w\|^2, \qquad i \in \{1, \dots, N\}.$$

The goal is to make agents agree on a common solution $(w^\star, b^\star)$, so that all of them can compute the separating hyperplane as $\{z \in \mathbb{R}^d \mid (w^\star)^\top z + b^\star = 0\}$.

Data generation model

We consider a bidimensional sample space ($d = 2$). Agents generate a certain number of samples (between 2 and 5) for both labels. For each label, the samples are drawn according to a multivariate gaussian distribution, with covariance matrix equal to the identity and mean equal to $(0, 0)$ (for the label 1) and $(3, 2)$ (for the label -1). The regularization parameter is set to $C = 10$.

Complete code

Listing 27: examples/setups/logistic_regression/launcher.py

```
#################################################################
# COST-COUPLED Example
# Logistic Regression for Classification
#
# Each agent has a certain number of randomly generated points, labeled 1 or -1.
# The points are generated by agents according to a multivariate normal distribution,
# with different mean and covariance for the two labels.
#################################################################
# Compared Algorithms:
#
# - Distributed Subgradient
# - Gradient Tracking
#################################################################

import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import SubgradientMethod, GradientTracking, DualDecomposition
from disropt.functions import Variable, SquaredNorm, Logistic
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings
from disropt.problems import Problem

# get MPI info
NN = MPI.COMM_WORLD.Get_size()
agent_id = MPI.COMM_WORLD.Get_rank()

# Generate a common graph (everyone uses the same seed)
Adj = binomial_random_graph(NN, p=0.3, seed=1)
W = metropolis_hastings(Adj)

np.random.seed(10*agent_id)

######################
# Problem parameters
######################

# parameters of gaussians
mu = (np.array([0, 0]).transpose(), np.array([3, 2]).transpose())
sigma = (np.eye(2), np.eye(2))
```

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```
dim = mu[0].shape[0]  # dimension of sample space

# number of samples (min 2 max 5 for each label)
nsamp = (np.random.randint(2, 6), np.random.randint(2, 6))

# regularization parameter
C = 10

######################
# Generate problem data
######################

# points
points = np.zeros((dim, nsamp[0]+nsamp[1]))
points[:, 0:nsamp[0]] = np.random.multivariate_normal(mu[0], sigma[0], nsamp[0]).
↪transpose()
points[:, nsamp[0]:] = np.random.multivariate_normal(mu[1], sigma[1], nsamp[1]).
↪transpose()

# labels
labels = np.ones((sum(nsamp), 1))
labels[nsamp[0]:] = -labels[nsamp[0]:]

# cost function
z = Variable(dim+1)
A = np.ones((dim+1, 1))
A[-1] = 0
obj_func = (C / (2 * NN)) * SquaredNorm(A @ z)

for j in range(sum(nsamp)):
    e_j = np.zeros((sum(nsamp), 1))
    e_j[j] = 1
    A_j = np.vstack((points @ e_j, 1))
    obj_func += Logistic(- labels[j] * A_j @ z)

######################
# Distributed algorithms
######################

# local agent and problem
agent = Agent(
    in_neighbors=np.nonzero(Adj[agent_id, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, agent_id])[0].tolist(),
    in_weights=W[agent_id, :].tolist())
pb = Problem(obj_func)
agent.set_problem(pb)

# instantiate the algorithms
x0 = 5*np.random.rand(dim+1, 1)

subgr = SubgradientMethod(agent=agent,
                          initial_condition=x0,
```

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```
                          enable_log=True)

gradtr = GradientTracking(agent=agent,
                          initial_condition=x0,
                          enable_log=True)

def step_gen(k):
    return 1/((k+1)**0.6)

constant_stepsize = 0.001
num_iterations    = 20000

# run the algorithms
subgr_seq      = subgr.run(iterations=num_iterations, stepsize=step_gen)
gradtr_seq     = gradtr.run(iterations=num_iterations, stepsize=constant_stepsize)

# print results
print("Subgradient method: agent {}: {}".format(agent_id, subgr.get_result().flatten()))
print("Gradient tracking: agent {}: {}".format(agent_id, gradtr.get_result().flatten()))

# save information
if agent_id == 0:
    with open('info.pkl', 'wb') as output:
        pickle.dump({'N': NN, 'size': dim+1, 'iterations': num_iterations}, output,␣
↪pickle.HIGHEST_PROTOCOL)

with open('agent_{}_func.pkl'.format(agent_id), 'wb') as output:
    pickle.dump(obj_func, output, pickle.HIGHEST_PROTOCOL)

np.save("agent_{}_seq_subgr.npy".format(agent_id), np.squeeze(subgr_seq))
np.save("agent_{}_seq_gradtr.npy".format(agent_id), np.squeeze(gradtr_seq))
```

Listing 28: examples/setups/logistic_regression/results.py

```
import numpy as np
import matplotlib.pyplot as plt
from disropt.problems import Problem
import pickle

# initialize
with open('info.pkl', 'rb') as inp:
    info = pickle.load(inp)
NN    = info['N']
iters = info['iterations']
size  = info['size']

# load agent data
seq_subgr   = np.zeros((NN, iters, size))
seq_gradtr  = np.zeros((NN, iters, size))
local_function = {}
for i in range(NN):
    seq_subgr[i,:,:]   = np.load("agent_{}_seq_subgr.npy".format(i))
```

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```
    seq_gradtr[i,:,:]  = np.load("agent_{}_seq_gradtr.npy".format(i))
    with open('agent_{}_func.pkl'.format(i), 'rb') as inp:
        local_function[i] = pickle.load(inp)

# solve centralized problem
global_obj_func = 0
for i in range(NN):
    global_obj_func += local_function[i]

global_pb = Problem(global_obj_func)
x_centr = global_pb.solve()
cost_centr = global_obj_func.eval(x_centr)
x_centr = x_centr.flatten()

# compute cost errors
cost_err_subgr   = np.zeros((NN, iters))
cost_err_gradtr  = np.zeros((NN, iters))

for i in range(NN):
    for t in range(iters):
        # first compute global function value at local point
        cost_ii_tt_subgr   = 0
        cost_ii_tt_gradtr  = 0
        for j in range(NN):
            cost_ii_tt_subgr   += local_function[j].eval(seq_subgr[i, t, :][:, None])
            cost_ii_tt_gradtr  += local_function[j].eval(seq_gradtr[i, t, :][:, None])

        # then compute errors
        cost_err_subgr[i, t]   = abs(cost_ii_tt_subgr - cost_centr)
        cost_err_gradtr[i, t]  = abs(cost_ii_tt_gradtr - cost_centr)

# plot maximum cost error
plt.figure()
plt.title('Maximum cost error (among agents)')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$\max_{i} \: \left|(\sum_{j=1}^N f_j(x_i^k) - f^\star)/f^\star \right|$")
plt.semilogy(np.arange(iters), np.amax(cost_err_subgr/cost_centr, axis=0), label=
↪'Distributed Subgradient')
plt.semilogy(np.arange(iters), np.amax(cost_err_gradtr/cost_centr, axis=0), label=
↪'Gradient Tracking')
plt.legend()

# plot maximum solution error
sol_err_subgr   = np.linalg.norm(seq_subgr - x_centr[None, None, :], axis=2)
sol_err_gradtr  = np.linalg.norm(seq_gradtr - x_centr[None, None, :], axis=2)

plt.figure()
plt.title('Maximum solution error (among agents)')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$\max_{i} \: \|x_i^k - x^\star\|$")
plt.semilogy(np.arange(iters), np.amax(sol_err_subgr, axis=0), label='Distributed␣
↪Subgradient')
```

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```
plt.semilogy(np.arange(iters), np.amax(sol_err_gradtr, axis=0), label='Gradient Tracking
↪')
plt.legend()

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 20 --oversubscribe python launcher.py
> python results.py
```

References

4.2.2 Common-cost: classification via Support Vector Machine

For the *common-cost* set-up, we consider a classification scenario [NoBü11]. In this example, a linear model is trained by maximizing the distance of the separating hyperplane from the training points. The complete code of this example is given *at the end of this page*.

Problem formulation

Suppose there are N agents, where each agent i is equipped with m_i points $p_{i,1}, \dots, p_{i,m_i} \in \mathbb{R}^d$ (which represent training samples in a d-dimensional feature space). Moreover, suppose the points are associated to binary labels, that is, each point $p_{i,j}$ is labeled with $\ell_{i,j} \in \{-1, 1\}$, for all $j \in \{1, \dots, m_i\}$ and $i \in \{1, \dots, N\}$. The problem consists of building a classification model from the training samples. In particular, we look for a separating hyperplane of the form $\{z \in \mathbb{R}^d \mid w^\top z + b = 0\}$ such that it separates all the points with $\ell_i = -1$ from all the points with $\ell_i = 1$.

In order to maximize the distance of the separating hyperplane from the training points, one can solve the following (convex) quadratic program:

$$\begin{aligned} \min_{w,b,\xi} \ & \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\sum_{j=1}^{m_i}\xi_{i,j} \\ \text{subject to } & \ell_{i,j}(w^\top p_{i,j} + b) \geq 1 - \xi_{i,j}, \qquad \forall\, j, i \\ & \xi \geq 0, \end{aligned}$$

where $C > 0$ is a parameter affecting regularization. The optimization problem above is called *soft-margin SVM* since it allows for the presence of outliers by activating the variables $\xi_{i,j}$ in case a separating hyperplane does not exist. Notice that the problem can be viewed either as a common-cost problem or as a cost-coupled problem(refer to the *general formulations*). Here we consider the problem as a common cost, with the common objective function equal to

$$f(w, b, \xi) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\sum_{j=1}^{m_i}\xi_{i,j}$$

and each local constraint set X_i given by

$$X_i = \{(w, b, \xi) \mid \xi \geq 0, \ell_{i,j}(w^\top p_{i,j} + b) \geq 1 - \xi_{i,j}, \text{ for all } j \in \{1, \dots, m_i\}\}.$$

The goal is to make agents agree on a common solution $(w^\star, b^\star, \xi^\star)$, so that all of them can compute the soft-margin separating hyperplane as $\{z \in \mathbb{R}^d \mid (w^\star)^\top z + b^\star = 0\}$.

Data generation model

We consider a bidimensional sample space ($d = 2$). Agents generate a certain number of samples (between 2 and 5) for both labels. For each label, the samples are drawn according to a multivariate gaussian distribution, with covariance matrix equal to the identity and mean equal to $(0, 0)$ (for the label 1) and $(3, 2)$ (for the label -1). The regularization parameter is set to $C = 10$.

Complete code

Listing 29: examples/setups/svm/launcher.py

```
#################################################################
# COMMON-COST Example
# Support Vector Machine
#
# Each agent has a certain number of randomly generated points, labeled 1 or -1.
# The points are generated by agents according to a multivariate normal distribution,
# with different mean and covariance for the two labels.
#
#################################################################
# Executed Algorithm: Constraints Consensus
#################################################################

import dill as pickle
import numpy as np
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import ConstraintsConsensus
from disropt.functions import Variable, SquaredNorm
from disropt.utils.graph_constructor import binomial_random_graph
from disropt.problems import Problem

# get MPI info
NN = MPI.COMM_WORLD.Get_size()
agent_id = MPI.COMM_WORLD.Get_rank()

# Generate a common graph (everyone uses the same seed)
Adj = binomial_random_graph(NN, p=0.03, seed=1)

np.random.seed(10*agent_id)
######################
# Problem parameters
######################

# parameters of gaussians
mu = (np.array([0, 0]).transpose(), np.array([3, 2]).transpose())
sigma = (np.eye(2), np.eye(2))

dim = mu[0].shape[0]  # dimension of sample space

# number of samples (min 2 max 5 for each label)
nsamp = (np.random.randint(2, 6), np.random.randint(2, 6))
```

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```
# regularization parameter
C = 10

######################
# Generate problem data
######################

# points
points = np.zeros((dim, nsamp[0]+nsamp[1]))
points[:, 0:nsamp[0]] = np.random.multivariate_normal(mu[0], sigma[0], nsamp[0]).
↪transpose()
points[:, nsamp[0]:] = np.random.multivariate_normal(mu[1], sigma[1], nsamp[1]).
↪transpose()

# labels
labels = np.ones((sum(nsamp), 1))
labels[nsamp[0]:] = -labels[nsamp[0]:]

# cost function
z = Variable(dim+1+NN)
A = np.zeros((dim+1+NN, dim))
A[0:dim:, :] = np.eye(dim)  # w = A @ z
B = np.zeros((dim+1+NN, 1))
B[dim+1:dim+NN+1] = np.ones((NN, 1))  # xi_1 + ... + xi_N = B @ z
D = np.zeros((dim+1+NN, 1))
D[dim] = 1  # b = D @ z
E = np.zeros((dim+1+NN, 1))
E[dim+1+agent_id] = 1  # xi_i = E @ z

obj_func = (1/2) * (A @ z) @ (A @ z) + C * (B @ z)

# constraints
F = np.zeros((dim+1+NN, NN))
F[dim+1:dim+NN+1:, :] = np.eye(NN)

constr = []
for idx in np.arange(F.shape[1]):
    constr.append(F[:, idx][:, None] @ z >= 0)

for j in range(sum(nsamp)):
    constr.append(float(labels[j]) * (points[:, j].reshape(2, 1) @ (A @ z) + D @ z) >= 1␣
↪- E @ z)  # j-th point

######################
# Distributed algorithms
######################

# local agent and problem
agent = Agent(
    in_neighbors=np.nonzero(Adj[agent_id, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, agent_id])[0].tolist())
```

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```
pb = Problem(obj_func, constr)
agent.set_problem(pb)
# instantiate the algorithm
constrcons = ConstraintsConsensus(agent=agent,
                                  enable_log=True)

n_iter = NN*3

# run the algorithm
constrcons_seq = constrcons.run(iterations=n_iter, verbose=True)

# print results
constrcons_x = constrcons.get_result()

print("Agent {}: {}".format(agent_id, constrcons_x.flatten()))  # save information

if agent_id == 0:
    with open('info.pkl', 'wb') as output:
        pickle.dump({'N': NN, 'size': NN+dim+1, 'iterations': n_iter}, output, pickle.
↪HIGHEST_PROTOCOL)
    with open('objective_function.pkl', 'wb') as output:
        pickle.dump(obj_func, output, pickle.HIGHEST_PROTOCOL)

with open('agent_{}_constr.pkl'.format(agent_id), 'wb') as output:
    pickle.dump(constr, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_seq.npy".format(agent_id), constrcons_seq)
```

Listing 30: examples/setups/svm/results.py

```
import numpy as np
import matplotlib.pyplot as plt
from disropt.problems import Problem
import pickle
import tikzplotlib

# initialize
with open('info.pkl', 'rb') as inp:
    info = pickle.load(inp)
NN = info['N']
iters = info['iterations']
size = info['size']

# load agent data
sequence = np.zeros((NN, iters, size))
local_constr = {}
for i in range(NN):
    sequence[i, :, :] = np.load("agent_{}_seq.npy".format(i), allow_pickle=True).
↪reshape((iters, size))
    with open('agent_{}_constr.pkl'.format(i), 'rb') as inp:
        local_constr[i] = pickle.load(inp)
with open('objective_function.pkl', 'rb') as inp:
    obj_func = pickle.load(inp)
```

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```
# solve centralized problem
global_constr = []
for i in range(NN):
    global_constr.extend(local_constr[i])
global_pb = Problem(obj_func, global_constr)
x_centr = global_pb.solve()
cost_centr = obj_func.eval(x_centr)

# compute cost errors
cost_err = np.zeros((NN, iters))

for i in range(NN):
    for t in range(iters):
        cost_err[i, t] = abs(obj_func.eval(sequence[i, t, :].reshape((size, 1))) - cost_
↪centr)

# compute max violation
vio_err = np.zeros((NN, iters))
for i in range(NN):
    for t in range(iters):
        xt = sequence[i, t, :].reshape((size, 1))
        max_err = np.zeros((len(global_constr), 1))
        for c in range(len(global_constr)):
            max_err[c] = global_constr[c].function.eval(xt)
        vio_err[i, t] = np.max(max_err)

# Plot the evolution of the local estimates
# generate N colors
colors = {}
for i in range(NN):
    colors[i] = np.random.rand(3, 1).flatten()

# plot cost error
plt.figure()
plt.title('Evolution of cost error')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$|f(x_i^k) - f^\star|$")

for i in range(NN):
    plt.plot(np.arange(iters), cost_err[i, :], color=colors[i])

# # plot violation error
plt.figure()
plt.title('Maximum constraint violation')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$max_j g(x_i^k)$")

for i in range(NN):
    plt.plot(np.arange(iters), vio_err[i, :], color=colors[i])

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 30 --oversubscribe python launcher.py
> python results.py
```

References

4.2.3 Constraint-coupled: charging of Plug-in Electric Vehicles (PEVs)

For the *constraint-coupled* set-up, we consider the problem of determining an optimal overnight charging schedule for a fleet of Plug-in Electric Vehicles (PEVs) [FalMa17]. The model described in this page is inspired by the model in the paper [VuEs16] (we consider the "charge-only" case without the integer constraints on the input variables). The complete code of this example is given *at the end of this page*.

Problem formulation

Suppose there is a fleet of N PEVs (agents) that must be charged by drawing power from the same electricity distribution network. Assuming the vehicles are connected to the grid at a certain time (e.g., at midnight), the goal is to determine an optimal overnight schedule to charge the vehicles, since the electricity price varies during the charging period.

Formally, we divide the entire charging period into a total of $T = 24$ time slots, each one of duration $\Delta T = 20$ minutes. For each PEV $i \in \{1, \dots, N\}$, the charging power at time step k is equal to $P_i u_i(k)$, where $u_i(k) \in [0, 1]$ is the input to the system and P_i is the maximum charging power that can be fed to the i-th vehicle.

System model

The state of charge of the i-th battery is denoted by $e_i(k)$, its initial state of charge is E_i^{init}, which by the end of the charging period has to attain at least E_i^{ref}. The charging conversion efficiency is denoted by $\zeta_i^{\text{u}} \triangleq 1 - \zeta_i$, where $\zeta_i > 0$ encodes the conversion losses. The battery's capacity limits are denoted by $E_i^{\min}, E_i^{\max} \geq 0$. The system's dynamics are therefore given by

$$
\begin{aligned}
& e_i(0) = E_i^{\text{init}} \\
& e_i(k+1) = e_i(k) + P_i \Delta T \zeta_i^u u_i(k), \qquad k \in \{0, \dots, T-1\} \\
& e_i(T) \geq E_i^{\text{ref}} \\
& E_i^{\min} \leq e_i(k) \leq E_i^{\max}, \qquad\qquad k \in \{1, \dots, T\} \\
& u_i(k) \in [0, 1], \qquad\qquad k \in \{0, \dots, T-1\}.
\end{aligned}
$$

To model congestion avoidance of the power grid, we further consider the following (linear) coupling constraints among all the variables

$$
\sum_{i=1}^{N} P_i u_i(k) \leq P^{\max}, \qquad k \in \{0, \dots, T-1\},
$$

where $P^{\max}$ is the maximum power that the be drawn from the grid.

Optimization problem

We assume that, at each time slot k, electricity has unit cost equal to $C^{\mathrm{u}}(k)$. Since the goal is to minimize the overall consumed energy price, the global optimization problem can be posed as

$$\begin{aligned} \min_{u,e} \quad & \sum_{i=1}^{N} \sum_{k=0}^{T-1} C^{\mathrm{u}}(k) P_i u_i(k) \\ \text{subject to} \quad & \sum_{i=1}^{N} P_i u_i(k) \leq P^{\max}, \qquad k \in \{0, \dots, T-1\} \\ & (u_i, e_i) \in X_i, \qquad i \in \{1, \dots, N\}. \end{aligned}$$

The problem is recognized to be a *constraint-coupled* problem, with local variables x_i equal to the stack of $e_i(k), u_i(k)$ for $k \in \{0, \dots, T-1\}$, plus $e_i(T)$. The local objective function is equal to

$$f_i(x_i) = \sum_{k=0}^{T-1} P_i u_i(k) C^{\mathrm{u}}(k),$$

the local constraint set is equal to

$$X_i = \{(e_i, u_i) \in \mathbb{R}^{T+1} \times \mathbb{R}^T \text{ such that local dynamics is satisfied}\}$$

and the local coupling constraint function $g_i : \mathbb{R}^{2T+1} \to \mathbb{R}^T$ has components

$$g_{i,k}(x_i) = P_i u_i(k) - \frac{P^{\max}}{N}, \qquad k \in \{0, \dots, T-1\}.$$

The goal is to make each agent compute its portion $x_i^\star = (e_i^\star, u_i^\star)$ of an optimal solution $(x_1^\star, \dots, x_N^\star)$ of the optimization problem, so that all of them can know their own assignment of the optimal charging schedule, given by $(u_i^\star(0), \dots, u_i^\star(T-1))$.

Data generation model

The data are generated according to table in [VuEs16] (see Appendix).

Complete code

Listing 31: examples/setups/pev/launcher.py

```
################################################################
# CONSTRAINT-COUPLED Example
# Charging of Plug-in Electric Vehicles (PEVs)
#
# The problem consists of finding an optimal overnight schedule to
# charge electric vehicles. See [Vu16] for the problem model
# and generation parameters.
#
# Note: in this example we consider the "charging-only" case
################################################################
# Compared Algorithms:
#
# - Distributed Dual Subgradient
```

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```
# - Distributed Primal Decomposition
#############################################################

import dill as pickle
import numpy as np
from numpy.random import uniform as rnd
from mpi4py import MPI
from disropt.agents import Agent
from disropt.algorithms import DualSubgradientMethod, PrimalDecomposition
from disropt.functions import Variable
from disropt.utils.graph_constructor import binomial_random_graph, metropolis_hastings,␣
↪binomial_random_graph_sequence
from disropt.problems import ConstraintCoupledProblem

# get MPI info
NN = MPI.COMM_WORLD.Get_size()
agent_id = MPI.COMM_WORLD.Get_rank()

# Generate a common graph (everyone uses the same seed)
Adj = binomial_random_graph(NN, p=0.2, seed=1)
W = metropolis_hastings(Adj)

# generate edge probabilities
edge_prob = np.random.uniform(0.3, 0.9, (NN, NN))
edge_prob[Adj == 0] = 0
i_lower = np.tril_indices(NN)
edge_prob[i_lower] = edge_prob.T[i_lower] # symmetrize

######################
# Generate problem parameters
######################

# Problem parameters are generated according to the table in [Vu16]

#### Common parameters

TT = 24 # number of time windows
DeltaT = 20 # minutes
# PP_max = 3 * NN # kWh
PP_max = 0.5 * NN # kWh
CC_u = rnd(19,35, (TT, 1)) # EUR/MWh - TT entries

#### Individual parameters

np.random.seed(10*agent_id)

PP = rnd(3,5) # kW
EE_min = 1 # kWh
EE_max = rnd(8,16) # kWh
EE_init = rnd(0.2,0.5) * EE_max # kWh
EE_ref = rnd(0.55,0.8) * EE_max # kWh
zeta_u = 1 - rnd(0.015, 0.075) # pure number
```

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```
######################
# Generate problem object
######################

# normalize unit measures
DeltaT = DeltaT/60 # minutes  -> hours
CC_u = CC_u/1e3    # Euro/MWh -> Euro/KWh

# optimization variables
z = Variable(2*TT + 1) # stack of e (state of charge) and u (input charging power)
e = np.vstack((np.eye(TT+1), np.zeros((TT, TT+1)))) @ z # T+1 components (from 0 to T)
u = np.vstack((np.zeros((TT+1, TT)), np.eye(TT))) @ z   # T components (from 0 to T-1)

# objective function
obj_func = PP * (CC_u @ u)

# coupling function
coupling_func = PP*u - (PP_max/NN)

# local constraints
e_0 = np.zeros((TT+1, 1))
e_T = np.zeros((TT+1, 1))
e_0[0] = 1
e_T[TT] = 1
constr = [e_0 @ e == EE_init, e_T @ e >= EE_ref] # feedback and reference constraints

for kk in range(0, TT):
    e_cur = np.zeros((TT+1, 1))
    u_cur = np.zeros((TT, 1))
    e_new = np.zeros((TT+1, 1))
    e_cur[kk] = 1
    u_cur[kk] = 1
    e_new[kk+1] = 1
    constr.append(e_new @ e == e_cur @ e + PP*DeltaT*zeta_u*u_cur @ u) # dynamics
    constr.extend([u_cur @ u <= 1, u_cur @ u >= 0]) # input constraints
    constr.extend([e_new @ e <= EE_max, e_new @ e >= EE_min]) # state constraints

######################
# Distributed algorithms
######################

# local agent and problem
agent = Agent(
    in_neighbors=np.nonzero(Adj[agent_id, :])[0].tolist(),
    out_neighbors=np.nonzero(Adj[:, agent_id])[0].tolist(),
    in_weights=W[agent_id, :].tolist())

pb = ConstraintCoupledProblem(obj_func, constr, coupling_func)
agent.set_problem(pb)

# instantiate the algorithms
```

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```
# y0  = np.zeros((TT, 1))
# mu0 = np.zeros((TT, 1))
y0 = 10*np.random.rand(TT, 1)
mu0 = 10*np.random.rand(TT, 1)

theothers = [i for i in range(NN) if i != agent_id]
y_others = agent.communicator.neighbors_exchange(y0, theothers, theothers, False)
y_others[agent_id] = y0
y_mean = sum([x for _, x in y_others.items()])/NN
y0 -= y_mean

dds = DualSubgradientMethod(agent=agent,
                            initial_condition=mu0,
                            enable_log=True)

dpd = PrimalDecomposition  (agent=agent,
                            initial_condition=y0,
                            enable_log=True)

##########################################

num_iterations = 1000

# generate sequence of adjacency matrices
Adj_seq = binomial_random_graph_sequence(Adj, num_iterations, edge_prob, NN, 1)

def step_gen(k): # define a stepsize generator
    return 1/((k+1)**0.6)
# def step_gen(k): # define a stepsize generator
#     return 0.01

def update_graph(k):
    Adj_k = Adj_seq[k]
    W_k = metropolis_hastings(Adj_k)
    agent.set_neighbors(in_neighbors=np.nonzero(Adj_k[agent_id, :])[0].tolist(),
                        out_neighbors=np.nonzero(Adj_k[:, agent_id])[0].tolist())
    agent.set_weights(in_weights=W_k[agent_id, :].tolist())

# run the algorithms
if agent_id == 0:
    print("Distributed dual subgradient")
_, dds_seq = dds.run(iterations=num_iterations, stepsize=step_gen, verbose=True)

if agent_id == 0:
    print("Distributed primal decomposition")
dpd_seq, _, _ = dpd.run(iterations=num_iterations, stepsize=step_gen, M=30.0,␣
↪verbose=True)

# save information
if agent_id == 0:
    with open('info.pkl', 'wb') as output:
        pickle.dump({'N': NN, 'iterations': num_iterations, 'n_coupling': TT}, output,␣
↪pickle.HIGHEST_PROTOCOL)
```

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```
with open('agent_{}_objective_func.pkl'.format(agent_id), 'wb') as output:
    pickle.dump(obj_func, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_coupling_func.pkl'.format(agent_id), 'wb') as output:
    pickle.dump(coupling_func, output, pickle.HIGHEST_PROTOCOL)
with open('agent_{}_local_constr.pkl'.format(agent_id), 'wb') as output:
    pickle.dump(constr, output, pickle.HIGHEST_PROTOCOL)
np.save("agent_{}_seq_dds.npy".format(agent_id), dds_seq)
np.save("agent_{}_seq_dpd.npy".format(agent_id), dpd_seq)
```

Listing 32: examples/setups/pev/results.py

```
import numpy as np
import matplotlib.pyplot as plt
from disropt.problems import Problem
from disropt.functions import ExtendedFunction
from disropt.constraints import ExtendedConstraint
import dill as pickle

# initialize
with open('info.pkl', 'rb') as inp:
    info = pickle.load(inp)
NN    = info['N']
iters = info['iterations']
SS    = info['n_coupling']

# load agent data
seq_dds = {}
seq_dpd = {}
local_obj_func = {}
local_coup_func = {}
local_constr = {}
for i in range(NN):
    seq_dds[i] = np.load("agent_{}_seq_dds.npy".format(i))
    seq_dpd[i] = np.load("agent_{}_seq_dpd.npy".format(i))
    with open('agent_{}_objective_func.pkl'.format(i), 'rb') as inp:
        local_obj_func[i] = pickle.load(inp)
    with open('agent_{}_coupling_func.pkl'.format(i), 'rb') as inp:
        local_coup_func[i] = pickle.load(inp)
    with open('agent_{}_local_constr.pkl'.format(i), 'rb') as inp:
        local_constr[i] = pickle.load(inp)

# solve centralized problem
dim = sum([f.input_shape[0] for _, f in local_obj_func.items()]) # size of overall␣
↪variable
centr_obj_func = 0
centr_constr = []
centr_coupling_func = 0
pos = 0
for i in range(NN):
    n_i = local_obj_func[i].input_shape[0]
```

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```
    centr_obj_func += ExtendedFunction(local_obj_func[i], n_var = dim-n_i, pos=pos)
    centr_constr.extend(ExtendedConstraint(local_constr[i], n_var = dim-n_i, pos=pos))
    centr_coupling_func += ExtendedFunction(local_coup_func[i], n_var = dim-n_i, pos=pos)

    pos += n_i

centr_constr.append(centr_coupling_func <= 0)
global_pb = Problem(centr_obj_func, centr_constr)
x_centr = global_pb.solve()
cost_centr = centr_obj_func.eval(x_centr).flatten()
x_centr = x_centr.flatten()

# compute costs and coupling values
cost_seq_dds = np.zeros(iters) - cost_centr
cost_seq_dpd = np.zeros(iters) - cost_centr
coup_seq_dds = np.zeros((iters, SS))
coup_seq_dpd = np.zeros((iters, SS))

for t in range(iters):
    for i in range(NN):
        cost_seq_dds[t] += local_obj_func[i].eval(seq_dds[i][t, :])
        cost_seq_dpd[t] += local_obj_func[i].eval(seq_dpd[i][t, :])
        coup_seq_dds[t] += np.squeeze(local_coup_func[i].eval(seq_dds[i][t, :]))
        coup_seq_dpd[t] += np.squeeze(local_coup_func[i].eval(seq_dpd[i][t, :]))

# plot cost
plt.figure()
plt.title('Evolution of cost error')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$| (\sum_{i=1}^N f_i(x_i^k) - f^\star)/f^\star |$")
plt.semilogy(np.arange(iters), np.abs(cost_seq_dds/cost_centr), label='Distributed Dual␣
↪Subgradient')
plt.semilogy(np.arange(iters), np.abs(cost_seq_dpd/cost_centr), label='Distributed␣
↪Primal Decomposition')
plt.legend()

# plot maximum coupling constraint value
plt.figure()
plt.title('Evolution of maximum coupling constraint value')
plt.xlabel(r"iteration $k$")
plt.ylabel(r"$\max_{s} \: \sum_{i=1}^N g_{is}(x_i^k)$")
plt.plot(np.arange(iters), np.amax(coup_seq_dds, axis=1), label='Distributed Dual␣
↪Subgradient')
plt.plot(np.arange(iters), np.amax(coup_seq_dpd, axis=1), label='Distributed Primal␣
↪Decomposition')
plt.legend()

plt.show()
```

The two files can be executed by issuing the following commands in the example folder:

```
> mpirun -np 50 --oversubscribe python launcher.py
```

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```
> python results.py
```

References

CHAPTER

FIVE

API DOCUMENTATION

disropt has been designed to make the execution and implementation of distributed algorithms as easy as possible. Here you find the full documentation.

5.1 Agents

5.1.1 Agent

class disropt.agents.**Agent**(*in_neighbors=None*, *out_neighbors=None*, *communicator=None*, *in_weights=None*, *out_weights=None*, *auto_local=True*)

Bases: object

The Agent object represents an agent in a network with communication capabilities

Parameters

- **in_neighbors** (*list*) – list of agents from which communication is received
- **out_neighbors** (*list*) – list of agents to which information is send
- **communicator** (Communicator, *optional*) – a Communicator object used to perform communications (if none is provided, it is automatically set to MPICommunicator). Defaults to None.
- **in_weights** (*list or dict, optional*) – list or dict containing weights to assign to information coming from each in-neighbor. If a list is provided, it must have lenght equal to the number of agents in the network. If a dict is provided, it must have a key for each in-neighbor and, associated to it, the correspondig weight. Defaults to None, implies equal in_weights to in-neighbors.
- **out_weights** (*list or dict, optional*) – list or dict containing weights to assign to out-neighbor. If a list is provided, it must have lenght equal to the number of agents in the network. If a dict is provided, it must have a key for each out-neighbor and, associated to it, the correspondig weight. Defaults to None, implies equal in_weights to out-neighbors.
- **auto_local** (*bool, optional*) – If False the (in-)weight for the local agent must be provided. Otherwise it is set automatically, provided that the in_weights have sum in [0,1]. Defaults to True.

id

id of the Agent

Type int

in_neighbors

list of in-neighbors

Type list

out_neighbors

list of out-neighbors

Type list

in_weights

a dict containing weights to assign to information coming from each in-neighbor.

Type dict

out_weights

a dict containing weights to assign to out-neighbors.

Type dict

communicator

Communicator object used to perform communications.

Type *Communicator*

problem

Local optimization problem.

Type *Problem*

neighbors_exchange(*obj*, *dict_neigh=False*, *event=None*)

Exchange data with neighbors (synchronously). Send obj to the out-neighbors and receive received_obj from in-neighbors

Parameters

- **obj** (Any) – object to send
- **dict_neigh** – True if obj contains a dictionary with different objects for each neighbor. Defaults to False.

Returns a dictionary containing an object for each in-neighbor

Return type dict

neighbors_receive_asynchronous()

Receive data from in-neighbors (if any have been sent)

Returns a dictionary containing an object for each in-neighbor that has sent one

Return type dict

neighbors_send(*obj*)

Send data to out-neighbors

Parameters **obj** (Any) – object to send

set_neighbors(*in_neighbors*, *out_neighbors*)

Set in and out neighbors

Parameters

- **in_neighbors** (list) – list of agents from which communication is received
- **out_neighbors** (list) – list of agents to which information is send

set_problem(*problem*)

set the local optimization problem

Parameters **problem** (Problem) – Problem object

Raises TypeError – Input must be a Problem object

set_weights(*in_weights=None*, *out_weights=None*, *auto_local=True*)

Set in_weights to assign to in-neighbors and the one for agent itself.

Parameters

- **in_weights** (Union[list, dict, None]) – list or dict contatining in_weights to assign to information coming from each in-neighbor. If a list is provided, it must have lenght equal to the number of agents in the network. If a dict is provided, it must have a key for each in-neighbor and, associated to it, the correspondig weight. Defaults to None, implies equal in_weights to in-neighbors.
- **out_weights** (Union[list, dict, None]) – list or dict contatining in_weights to assign to out-neighbors. If a list is provided, it must have lenght equal to the number of agents in the network. If a dict is provided, it must have a key for each out-neighbor and, associated to it, the correspondig weight. Defaults to None, implies equal in_weights to out neighbors.
- **auto_local** (bool) – If False the weight for the local agent must be provided. Otherwise it is set automatically, provided that the in_weights have sum in [0,1]. Defaults to True.

Raises

- ValueError – If a dict is provided as argument, it must contain a key for each in-neighbor.
- ValueError – Input must be list or dict
- ValueError – If auto_local is not False, the provided in_weights must have sum in [0,1]

5.2 Communicators

5.2.1 Communicator

class disropt.communicators.**Communicator**

Communicator abstract class

neighbors_exchange(*send_obj*, *in_neighbors*, *out_neighbors*, *dict_neigh*, *stop_event*)

Return type Dict[int, Any]

neighbors_receive(*neighbors*, *stop_event*)

Return type Dict[int, Any]

neighbors_receive_asynchronous(*neighbors*)

Return type Dict[int, Any]

neighbors_send(*obj*, *neighbors*)

5.2.2 MPICommunicator

class disropt.communicators.**MPICommunicator**

Bases: *disropt.communicators.communicators.Communicator*

Communicator class that performs communications through MPI. Requires mpi4py.

comm

communication world

size

size of the network.

Type int

rank

rank of the processor

Type int

neighbors_exchange(*send_obj*, *in_neighbors*, *out_neighbors*, *dict_neigh=False*, *stop_event=None*)

Exchange information (synchronously) with neighbors.

Parameters

- **send_obj** (Any) – object to send
- **in_neighbors** (List[int]) – list of in-neighbors
- **out_neighbors** (List[int]) – list of out-neighbors
- **dict_neigh** (bool) – True if send_obj contains a dictionary with different objects for each neighbor. Defaults to False
- **stop_event** (Optional[Event]) – an Event object that is monitored during the execution. If the event is set, the function returns immediately. Defaults to None (does not wait upon any event)

Return type Dict[int, Any]

Returns data received by in-neighbors

neighbors_receive(*neighbors*, *stop_event=None*)

Receive data from neighbors (waits until data are received from all neighbors).

Parameters

- **neighbors** (List[int]) – list of in-neighbors
- **stop_event** (Optional[Event]) – an Event object that is monitored during the execution. If the event is set, the function returns immediately. Defaults to None (does not wait upon any event)

Return type Dict[int, Any]

Returns data received by in-neighbors

neighbors_receive_asynchronous(*neighbors*)

Receive data (if any) from neighbors.

Parameters **neighbors** (List[int]) – list of in-neighbors

Return type Dict[int, Any]

Returns data received by in-neighbors (if any)

neighbors_send(*obj*, *neighbors*)

Send data to neighbors.

Parameters

- **obj** (Any) – object to send
- **neighbors** (List[int]) – list of out-neighbors

5.3 Algorithms

5.3.1 Consensus Algorithms

Consensus

class disropt.algorithms.consensus.**Consensus**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Consensus Algorithm [OlSa07]

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \dots$

$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_j^k$$

where $x_i \in \mathbb{R}^n$. The weight matrix $W = [w_{ij}]$ should be doubly-stochastic in order to have convergence to the average of the local initial conditions. If W is row-stochastic convergence is still attained but at a different point. Other type of matrices can be used, but convergence is not guaranteed. Also time-varying graphs can be adopted.

Parameters

- **agent** (Agent) – agent to execute the algorithm
- **initial_condition** (*numpy.ndarray*) – initial condition
- **enable_log** (*bool*) – True for enabling log

agent

agent to execute the algorithm

Type *Agent*

x0

initial condition

Type numpy.ndarray

x

current value of the local solution

Type numpy.ndarray

shape

shape of the variable

Type tuple

x_neigh

dictionary containing the local solution of the (in-)neighbors

Type dict

enable_log

True for enabling log

Type bool

get_result()

Return the actual value of x

Returns value of x

Return type numpy.ndarray

iterate_run(***kwargs*)

Run a single iterate of the algorithm

run(*iterations=100*, *verbose=False*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

AsynchronousConsensus

class disropt.algorithms.consensus.**AsynchronousConsensus**(*agent*, *initial_condition*, *enable_log=False*, *force_sleep=False*, *maximum_sleep=0.01*, *sleep_type='random'*, *force_computation_time=False*, *maximum_computation_time=0.01*, *computation_time_type='random'*, *force_unreliable_links=False*, *link_failure_probability=0*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Asynchronous Consensus Algorithm

From the perspective of agent i the algorithm works as follows. When agent i gets awake it updates its local solution as

$$x_i \leftarrow \sum_{j \in \mathcal{N}_i} w_{ij} x_{j|i}$$

where $\mathcal{N}_i$ is the current set of in-neighbors and $x_{j|i}, j \in \mathcal{N}_i$ is the local copy of x_j available at node i (which can be outdated, due to asynchrony, computation time and link failures).

Parameters

- **agent** (*Agent*) – agent to execute the algorithm
- **initial_condition** (ndarray) – initial condition
- **enable_log** (bool) – True for enabling log. Defaults to False.

- **force_sleep** (bool) – True if one wanst to force sleep after the computation phase. Defaults to False.
- **maximum_sleep** (float) – Maximum allowed sleep. Defaults to 0.01.
- **sleep_type** (str) – Type of sleep time("constant", "random"). Defaults to "random".
- **force_computation_time** (bool) – True if one want sto force length computation phase. Defaults to False.
- **maximum_computation_time** (float) – Maximum allowed computation time. Defaults to 0.01.
- **computation_time_type** (str) – Type of computation time ("constant", "random"). Defaults to "random".
- **force_unreliable_links** (bool) – True if one wants to force unreliable links. Defaults to False.
- **link_failure_probability** (float) – Probability of incoming links failure. Defaults to 0.

agent

agent to execute the algorithm

Type *Agent*

x0

initial condition

Type numpy.ndarray

x

current value of the local solution

Type numpy.ndarray

shape

shape of the variable

Type tuple

x_neigh

dictionary containing the local solution of the (in-)neighbors

Type dict

enable_log

True for enabling log

Type bool

timestamp_sequence_awake

list of timestamps at which node get awake

Type list

timestamp_sequence_sleep

list of timestamps at which node go to sleep

Type list

force_sleep

True if one wanst to force sleep after the computation phase. Defaults to False.

maximum_sleep

Maximum allowed sleep. Defaults to 0.01.

sleep_type

Type of sleep time("constant", "random"). Defaults to "random".

force_computation_time

True if one want sto force length computation phase. Defaults to False.

maximum_computation_time

Maximum allowed computation time. Defaults to 0.01.

computation_time_type

Type of computation time ("constant", "random"). Defaults to "random".

force_unreliable_links

True if one wants to force unreliable links. Defaults to False.

link_failure_probability

Probability of incoming links failure. Defaults to 0.

get_result()

Return the actual value of x

Returns value of x

Return type numpy.ndarray

iterate_run(***kwargs*)

Run a single iterate

run(*running_time=5.0*)

Run the asynchronous consensus algorithm for a certain amount of time

Parameters **running_time** (float) – Total run time. Defaults to 5.0.

Returns timestamp_sequence_awake, timestamp_sequence_sleep, sequence

Return type tuple

BlockConsensus

class disropt.algorithms.consensus.**BlockConsensus**(*agent*, *initial_condition*, *enable_log=False*, *blocks_list=None*, *probabilities=None*, *awakening_probability=1.0*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Block-wise consensus [FaNo19]

At each iteration, the agent can update its local estimate or not at each iteration according to a certain probability (awakening_probability). From the perspective of agent i the algorithm works as follows. At iteration k if the agent is awake, it selects a random block ℓ_i^k of its local solution and updates

$$x_{i,\ell}^{k+1} = \begin{cases} \sum_{j\in\mathcal{N}_i} w_{ij} x_{j|i,\ell}^k & \text{if} \ell = \ell_i^k \\ x_{i,\ell}^k & \text{otherwise} \end{cases}$$

where $\mathcal{N}_i$ is the current set of in-neighbors and $x_{j|i}, j \in \mathcal{N}_i$ is the local copy of x_j available at node i and $x_{i,\ell}$ denotes the ℓ-th block of x_i. Otherwise $x_i^{k+1} = x_i^k$.

Parameters

- **agent** (*Agent*) – agent to execute the algorithm
- **initial_condition** (ndarray) – initial condition
- **enable_log** (bool) – True for enabling log
- **blocks_list** (Optional[List[Tuple]]) – the list of blocks (list of tuples)
- **probabilities** (Optional[List[float]]) – list of probabilities of drawing each block
- **awakening_probability** (float) – probability of getting awake at each iteration

get_result()

Return the actual value of x

Returns value of x

Return type numpy.ndarray

iterate_run(***kwargs*)

Run a single iterate of the algorithm

run(*iterations=100*, *verbose=False*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

PushSumConsensus

class disropt.algorithms.consensus.**PushSumConsensus**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Push-Sum Consensus Algorithm

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \ldots$

$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_j^k$$
$$y_i^{k+1} = \sum_{j=1}^{N} w_{ij} y_j^k$$
$$z_i^{k+1} = \frac{x_i^{k+1}}{y_i^{k+1}}$$

where $x_i \in \mathbb{R}^n$. The weight matrix $W = [w_{ij}]$ should be column-stochastic in order to let z_i^k converge to the average of the local initial conditions. Also time-varying graphs can be adopted.

Parameters

- **agent** (Agent) – agent to execute the algorithm

- **initial_condition** (*numpy.ndarray*) – initial condition
- **enable_log** (*bool*) – True for enabling log

agent

agent to execute the algorithm

Type *Agent*

z0

initial condition

Type numpy.ndarray

z

current value of the local solution

Type numpy.ndarray

shape

shape of the variable

Type tuple

x_neigh

dictionary containing the x values of the (in-)neighbors

Type dict

y_neigh

dictionary containing the y values of the (in-)neighbors

Type dict

enable_log

True for enabling log

Type bool

get_result()

Return the actual value of x

Returns value of x

Return type numpy.ndarray

iterate_run(***kwargs*)

Run a single iterate of the algorithm

run(*iterations=100*, *verbose=False*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

References

5.3.2 (Sub)gradient based Algorithms

Distributed projected (Sub)gradient Method

class disropt.algorithms.subgradient.**SubgradientMethod**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.consensus.Consensus*

Distributed projected (sub)gradient method.

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \ldots$

$$y_i^k = \sum_{j=1}^{N} w_{ij} x_j^k$$
$$x_i^{k+1} = \Pi_X[y_i^k - \alpha^k \tilde{\nabla} f_i(y_i^k)]$$

where $x_i, y_i \in \mathbb{R}^n$, $X \subseteq \mathbb{R}^n$, α^k is a positive stepsize, w_{ij} denotes the weight assigned by agent i to agent j, $\Pi_X[]$ denotes the projection operator over the set X and $\tilde{\nabla} f_i(y_i^k) \in \partial f_i(y_i^k)$ a (sub)gradient of f_i computed at y_i^k. The weight matrix $W = [w_{ij}]_{i,j=1}^N$ should be doubly stochastic.

The algorithm, as written above, was originally presented in [NeOz09]. Many other variants and extension has been proposed, allowing for stochastic objective functions, time-varying graphs, local stepsize sequences. All these variant can be implemented through the *SubgradientMethod* class.

run(*iterations=1000*, *stepsize=0.001*, *verbose=False*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. If a function is given, it must take an iteration k as input and output the corresponding stepsize.. Defaults to 0.1.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float or a callable
- **ValueError** – Only sets (children of AstractSet) with explicit projections are currently supported
- **ValueError** – Only one constraint per time is currently supported

Return type ndarray

Returns return the sequence of estimates if enable_log=True.

Randomized Block (Sub)gradient Method

class disropt.algorithms.subgradient.**BlockSubgradientMethod**(*agent*, *initial_condition*, ***kwargs*)

Bases: *disropt.algorithms.consensus.BlockConsensus*

Distributed block subgradient method. This is a special instance of the Block Proximal Method in [FaNo19]

At each iteration, the agent can update its local estimate or not at each iteration according to a certain probability (awakening_probability). From the perspective of agent i the algorithm works as follows. At iteration k if the agent is awake, it selects a random block ℓ_i^k of its local solution and updates

$$y_i^k = \sum_{j \in \mathcal{N}_i} w_{ij} x_{j|i}^k \tag{5.1}$$

$$x_{i,\ell}^{k+1} = \begin{cases} \Pi_{X_\ell}\left[y_i^k - \alpha_i^k [\tilde{\nabla} f_i(y_i^k)]_\ell\right] & \text{if } \ell = \ell_i^k \\ x_{i,\ell}^k & \text{otherwise} \end{cases} \tag{5.2}$$

then it broadcasts $x_{i,\ell_i^k}^{k+1}$ to its out-neighbors. Otherwise (if the agent is not awake) $x_i^{k+1} = x_i^k$. Here $\mathcal{N}_i$ is the current set of in-neighbors and $x_{j|i}, j \in \mathcal{N}_i$ is the local copy of x_j available at node i and $x_{i,\ell}$ denotes the ℓ-th block of x_i. The weight matrix $W = [w_{ij}]_{i,j=1}^N$ should be doubly stochastic.

Notice that if there is only one block and awakening_probability=1 the *BlockSubgradientMethod* reduces to the *SubgradientMethod*.

run(*iterations=1000*, *stepsize=0.1*, *verbose=False*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. If a function is given, it must take an iteration k as input and output the corresponding stepsize.. Defaults to 0.1.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float or a callable
- **ValueError** – Only sets (children of AstractSet) with explicit projections are currently supported
- **ValueError** – Only one constraint per time is currently supported

Return type ndarray

Returns return the sequence of estimates if enable_log=True.

Distributed Gradient Tracking

class disropt.algorithms.gradient_tracking.**GradientTracking**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Gradient Tracking Algorithm [...]_

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \ldots$

$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_j^k - \alpha d_i^k$$

$$d_i^{k+1} = \sum_{j=1}^{N} w_{ij} d_j^k - [\nabla f_i(x_i^{k+1}) - \nabla f_i(x_i^k)]$$

where $x_i \in \mathbb{R}^n$ and $d_i \in \mathbb{R}^n$. The weight matrix $W = [w_{ij}]$ must be doubly-stochastic. Extensions to other class of weight matrices W are not currently supported.

Parameters

- **agent** (Agent) – agent to execute the algorithm
- **initial_condition** (*numpy.ndarray*) – initial condition for x_i
- **enable_log** (*bool*) – True for enabling log

agent

agent to execute the algorithm

Type *Agent*

x0

initial condition

Type numpy.ndarray

x

current value of the local solution

Type numpy.ndarray

d

current value of the local tracker

Type numpy.ndarray

shape

shape of the variable

Type tuple

x_neigh

dictionary containing the local solution of the (in-)neighbors

Type dict

d_neigh

dictionary containing the local tracker of the (in-)neighbors

Type dict

enable_log

True for enabling log

Type bool

get_result()

Return the actual value of x

Returns value of x

Return type numpy.ndarray

iterate_run(*stepsize*, ***kwargs*)

Run a single iterate of the gradient tracking algorithm

run(*iterations=1000*, *stepsize=0.1*, *verbose=False*)

Run the gradient tracking algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. Default is 0.01.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float

Return type ndarray

Returns return the sequence of estimates if enable_log=True.

Distributed Gradient Tracking (over directed, unbalanced graphs)

class disropt.algorithms.gradient_tracking.**DirectedGradientTracking**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.consensus.PushSumConsensus*

Gradient Tracking Algorithm [XiKh18]

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \dots$

$$x_i^{k+1} = \sum_{j=1}^{N} a_{ij} x_j^k - \alpha y_i^k$$

$$y_i^{k+1} = \sum_{j=1}^{N} b_{ij} (y_j^k - [\nabla f_j(x_j^{k+1}) - \nabla f_j(x_j^k)])$$

where $x_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}^n$. The weight matrix $A = [a_{ij}]$ must be row-stochastic, while $B = [b_{ij}]$ must be column-stochastic. The underlying graph can be directed (and unbalanced).

get_result()

Return the actual value of x

Returns value of x

Return type numpy.ndarray

iterate_run(***kwargs*)

Run a single iterate of the algorithm

References

5.3.3 Dual and Primal/Dual algorithms

Distributed dual decomposition

class disropt.algorithms.dual_decomp.**DualDecomposition**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Distributed dual decomposition.

From the perspective of agent i the algorithm works as follows.

Initialization: λ_{ij}^0 for all $j \in \mathcal{N}_i$

For $k = 0, 1, \ldots$

- Gather λ_{ji}^k from neighbors $j \in \mathcal{N}_i$
- Compute x_i^k as an optimal solution of

$$\min_{x_i \in X_i} \; f_i(x_i) + x_i^\top \sum_{j \in \mathcal{N}_i} (\lambda_{ij}^k - \lambda_{ji}^k)$$

- Gather x_j^k from neighbors $j \in \mathcal{N}_i$
- Update for all $j \in \mathcal{N}_i$

$$\lambda_{ij}^{k+1} = \lambda_{ij}^k + \alpha^k (x_i^k - x_j^k)$$

where $x_i \in \mathbb{R}^n$, $\lambda_{ij} \in \mathbb{R}^n$ for all $j \in \mathcal{N}_i$, $X_i \subseteq \mathbb{R}^n$ for all i and α^k is a positive stepsize.

The algorithm has been presented in ????.

get_result()

Return the current value primal solution and dual variable

Returns value of primal solution (np.ndarray), dual variables (dictionary of np.ndarray)

Return type tuple (primal, dual)

iterate_run(*stepsize*, ***kwargs*)

Run a single iterate of the algorithm

run(*iterations=1000*, *stepsize=0.1*, *verbose=False*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. If a function is given, it must take an iteration k as input and output the corresponding stepsize.. Defaults to 0.1.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float or a callable

Return type ndarray

Returns return a tuple (x, lambda) with the sequence of primal solutions and dual variables if enable_log=True.

Distributed ADMM

class disropt.algorithms.admm.**ADMM**(*agent*, *initial_lambda*, *initial_z*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Distributed ADMM.

From the perspective of agent i the algorithm works as follows.

Initialization: λ_{ij}^0 for all $j \in \mathcal{N}_i$, λ_{ii}^0 and z_i^0

For $k = 0, 1, \ldots$

- Compute x_i^k as the optimal solution of

$$\min_{x_i \in X_i} \; f_i(x_i) + x_i^\top \sum_{j \in \mathcal{N}_i \cup \{i\}} \lambda_{ij}^k + \frac{\rho}{2} \sum_{j \in \mathcal{N}_i \cup \{i\}} \|x_i - z_j^t\|^2$$

- Gather x_j^k and λ_{ji}^k from neighbors $j \in \mathcal{N}_i$
- Update z_i^{k+1}

$$z_i^{k+1} = \frac{\sum_{j \in \mathcal{N}_i \cup \{i\}} x_j^k}{|\mathcal{N}_i| + 1} + \frac{\sum_{j \in \mathcal{N}_i \cup \{i\}} \lambda_{ji}^k}{\rho(|\mathcal{N}_i| + 1)}$$

- Gather z_j^{k+1} from neighbors $j \in \mathcal{N}_i$
- Update for all $j \in \mathcal{N}_i$

$$\lambda_{ij}^{k+1} = \lambda_{ij}^k + \rho(x_i^k - z_j^{k+1})$$

where $x_i, z_i \in \mathbb{R}^n$, $\lambda_{ij} \in \mathbb{R}^n$ for all $j \in \mathcal{N}_i \cup \{i\}$, $X_i \subseteq \mathbb{R}^n$ for all i and ρ is a positive penalty parameter.

The algorithm has been presented in ????.

get_result()

Return the current value primal solution, dual variable and auxiliary primal variable

Returns value of primal solution (np.ndarray), dual variables (dictionary of np.ndarray), auxiliary primal variable (np.ndarray)

Return type tuple (primal, dual, auxiliary)

initialize_algorithm()

Initializes the algorithm

iterate_run(*rho*, ***kwargs*)

Run a single iterate of the algorithm

run(*iterations=1000*, *penalty=0.1*, *verbose=False*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **penalty** (float) – ADMM penalty parameter. Defaults to 0.1.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises TypeError – The number of iterations must be an int

Return type ndarray

Returns return a tuple (x, lambda, z) with the sequence of primal solutions, dual variables and auxiliary primal variables if enable_log=True.

Distributed dual subgradient method

class disropt.algorithms.dual_subgradient.**DualSubgradientMethod**(*agent*, *initial_condition*, *initial_runavg=None*, *enable_log=False*)

Bases: *disropt.algorithms.consensus.Consensus*

Distributed dual subgradient method.

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \dots$

$$
\begin{aligned}
y_i^k &= \sum_{j=1}^{N} w_{ij} \lambda_j^k \\
x_i^{k+1} &\in \arg\min_{x_i \in X_i} f_i(x_i) + g_i(x_i)^\top y_i^k \\
\lambda_i^{k+1} &= \Pi_{\lambda \geq 0}[y_i^k + \alpha^k g_i(x_i^{k+1})] \\
\hat{x}_i^{k+1} &= \hat{x}_i^k + \frac{\alpha^k}{\sum_{r=0}^{k} \alpha^r}(x_i^{k+1} - \hat{x}_i^k)
\end{aligned}
$$

where $x_i, \hat{x}_i \in \mathbb{R}^{n_i}$, $\lambda_i, y_i \in \mathbb{R}^S$, $X_i \subseteq \mathbb{R}^{n_i}$ for all i, α^k is a positive stepsize and $\Pi_{\lambda \geq 0}[]$ denotes the projection operator over the nonnegative orthant.

The algorithm has been presented in [FaMa17].

Warning: this algorithm is still under development

get_result()

Return the actual value of dual and primal averaged variable

Returns value of primal running average, value of dual variable

Return type tuple (dual, primal) of numpy.ndarray

run(*iterations=1000*, *stepsize=0.1*, *verbose=False*, *callback_iter=None*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. If a function is given, it must take an iteration k as input and output the corresponding stepsize.. Defaults to 0.1.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.
- **callback_iter** (Optional[Callable]) – callback function to be called at the end of each iteration. Must take an iteration k as input. Defaults to None.

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float or a callable

Return type ndarray

Returns return a tuple (lambda, x_hat) with the sequence of dual and primal estimates if enable_log=True.

Distributed Primal Decomposition

class disropt.algorithms.primal_decomp.**PrimalDecomposition**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Distributed primal decomposition.

From the perspective of agent i the algorithm works as follows.

Initialization: y_i^0 such that $\sum_{i=1}^N y_i^0 = 0$

For $k = 0, 1, \ldots$

- Compute $((x_i^k, \rho_i^k), \mu_i^k)$ as a primal-dual optimal solution of
- Gather μ_j^k from $j \in \mathcal{N}_i$ and update

$$y_i^{k+1} = y_i^k + \alpha^k \sum_{j \in \mathcal{N}_i} (\mu_i^k - \mu_j^k)$$

where $x_i \in \mathbb{R}^{n_i}$, $\mu_i, y_i \in \mathbb{R}^S$, $X_i \subseteq \mathbb{R}^{n_i}$ for all i and α^k is a positive stepsize.

The algorithm has been presented in ????.

get_result(*return_runavg=False*)

Return the current value of primal solution, allocation and cost

Parameters **return_runavg** (bool) – whether or not to return also running average of allocation. Defaults to False.

Returns value of primal solution, allocation, cost (if return_runavg = False) tuple (primal, allocation, allocation_avg cost) if return_runavg = True

Return type tuple (primal, allocation, cost) of numpy.ndarray

iterate_run(*stepsize*, *M*, *update_runavg*, *event*, ***kwargs*)

Run a single iterate of the algorithm

run(*iterations=1000*, *stepsize=0.1*, *M=1000.0*, *verbose=False*, *callback_iter=None*, *compute_runavg=False*, *runavg_start_iter=0*, *event=None*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. If a function is given, it must take an iteration k as input and output the corresponding stepsize.. Defaults to 0.1.
- **M** (float) – Value of the parameter M. Defaults to 1000.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.
- **callback_iter** (Optional[Callable]) – callback function to be called at the end of each iteration. Must take an iteration k as input. Defaults to None.
- **compute_runavg** (bool) – whether or not to compute also running average of allocation. Defaults to False.
- **runavg_start_iter** (int) – specifies when to start computing running average (applies only if compute_runavg = True). Defaults to 0.

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float or a callable
- **TypeError** – The parameter M must be a float

Return type ndarray

Returns return a tuple (x, y, J) with the sequence of primal solutionsm allocation estimates and cost if enable_log=True. If compute_runavg=True, then return (x, y, y_avg, J)

Distributed Primal Decomposition for MILPs

class disropt.algorithms.primal_decomp_milp.**PrimalDecompositionMILP**(*agent*, *initial_condition*, *enable_log=False*, *restriction=None*, *finite_time_add_restriction=0*)

Bases: *disropt.algorithms.primal_decomp.PrimalDecomposition*

Distributed primal decomposition for MILPs.

compute_restriction(*iterations*, *graph_diam*)

compute_violating_y(*allocation*)

iterate_run(*stepsize*, *M*, *update_runavg*, *event*, ***kwargs*)

Run a single iterate of the algorithm

mixed_integer_solution()

run(*n_agents*, *iterations=1000*, *stepsize=0.1*, *M=1000.0*, *verbose=False*, *callback_iter=None*, *fast_mode=True*, *max_cutting_planes=1000*, *milp_solver=None*, *extra_allocation=None*, *use_runavg=False*, *runavg_start_iter=0*, *event=None*, *max_consensus_iterations=None*, *max_consensus_graph_diam=None*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (int) – Number of iterations. Defaults to 1000.
- **stepsize** (Union[float, Callable]) – If a float is given as input, the stepsize is constant. If a function is given, it must take an iteration k as input and output the corresponding stepsize. Defaults to 0.1.
- **M** (float) – Value of the parameter M. Defaults to 1000.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.
- **callback_iter** (Optional[Callable]) – callback function to be called at the end of each iteration. Must take an iteration k as input. Defaults to None.
- **fast_mode** (bool) – If True, a mixed-integer solution is computed at each iteration, otherwise only at the last iteration. Defaults to True.
- **max_cutting_planes** (int) – maximum number of cutting planes for local solver. Defaults to 1000.
- **milp_solver** (Optional[str]) – MILP solver to use. Defaults to None (use default solver).

Raises

- **TypeError** – The number of iterations must be an int
- **TypeError** – The stepsize must be a float or a callable
- **TypeError** – The parameter M must be a float

Return type ndarray

Returns return a tuple (x, y) with the sequence of primal solutions and allocation estimates if enable_log=True. If fast_mode=True, the primal solutions are those of the convexified problem, otherwise they are the mixed-integer estimates.

ASYMM (beta)

class disropt.algorithms.asymm.**ASYMM**(*agent*, *graph_diameter*, *initial_condition*, *enable_log=False*, ***kwargs*)

Bases: *disropt.algorithms.misc.AsynchronousLogicAnd*

Asyncrhonous Distributed Method of Multipliers [FaGa19b]

See [FaGa19b] for the details.

> **Warning:** This algorithm is currently under development

dual_update_step()

get_result()

Return the value of the solution

primal_update_step()

reset_step()

Reset the matrix S and update e

run(*running_time=10.0*)

Run the algorithm

Parameters **maximum_running_time** – Maximum running time. Defaults to 1.

Raises TypeError – maximum running time must be a float

References

5.3.4 Set Membership Algorithms

Distributed Set Membership

class disropt.algorithms.setmembership.**SetMembership**(*agent*, *initial_condition*, *enable_log=False*)

Bases: *disropt.algorithms.consensus.Consensus*

Distributed Set Membership Algorithm [FaGa18]

From the perspective of agent i the algorithm works as follows. For $k = 0, 1, \ldots$

$$X_i^{k+1} = X_i^k \cap M_i^{k+1}$$
$$z_i^k = \sum_{j=1}^{N} w_{ij} x_j^k$$
$$x_i^{k+1} = \Pi_{X_i^{k+1}}[z_i^k]$$

where $x_i, z_i \in \mathbb{R}^n$, $M_i^{k+1}, X_i^{k+1} \subseteq \mathbb{R}^n$ are the current feasible (measurement) set and the feasible (parameter) set respectively, and $\Pi_X[]$ denotes the projection operator over the set X.

Parameters

- **agent** (Agent) – agent to execute the algorithm (must be a Agent)
- **initial_condition** (*numpy.ndarray*) – initial condition
- **enable_log** (*bool*) – True for enabling log

agent

agent to execute the algorithm

Type *Agent*

x0

initial condition

Type numpy.ndarray

x

current value of the local solution

Type numpy.ndarray

shape

shape of the variable

Type tuple

x_neigh

dictionary containing the local solution of the (in-)neighbors

Type dict

enable_log

True for enabling log

Type bool

measure()

Takes a new measurement and updates parameter_set

set_measure_generator(*generator*)

set the measure generator

Parameters **generator** (Callable) – measure generator

Asynchronous Distributed Set Membership

class disropt.algorithms.setmembership.**AsynchronousSetMembership**(*agent*, *initial_condition*, ***kwargs*)

Bases: *disropt.algorithms.consensus.AsynchronousConsensus*

Asynchronous Distributed Set Membership Algorithm [FaGa19]

Parameters

- **agent** (Agent) – agent to execute the algorithm (must be a Agent)
- **initial_condition** (*numpy.ndarray*) – initial condition
- **enable_log** (*bool*) – True for enabling log

agent

agent to execute the algorithm

Type *Agent*

x0

initial condition

Type numpy.ndarray

x

current value of the local solution

Type numpy.ndarray

shape

shape of the variable

Type tuple

x_neigh

dictionary containing the local solution of the (in-)neighbors

Type dict

enable_log
True for enabling log

Type bool

timestamp_sequence
list of timestamps

Type list

measure()
Takes a new measurement and updates parameter_set

set_measure_generator(*generator*)
set the measure generator

Parameters **generator** (Callable) – measure generator

References

5.3.5 Constraint Exchange Algorithms

Constraints Consensus

class disropt.algorithms.constraintexchange.**ConstraintsConsensus**(*agent*, *enable_log=False*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Constraints Consensus Algorithm [NoBu11]

This algorithm solves convex and abstract programs in the form

x
current value of the local solution

Type numpy.ndarray

B
basis associated to the local solution

Type numpy.ndarray

shape
shape of the variable

Type tuple

x_neigh
dictionary containing the local solution of the (in-)neighbors

Type dict

sequence_x
sequence of local solutions

Type numpy.ndarray

Parameters

- **agent** (Agent) – agent to execute the algorithm

- **enable_log** (*bool*) – True to enable log

compare_constr(*a*, *b*)

Compare two constraints to check whether they are equal

compute_basis(*constraints*)

Compute a (minimal) basis for the given constraint list

constr_to_dict(*constraints*)

Convert constraint list to dictionary

dict_to_constr(*dictio*)

Convert dictionary to constraint list

get_basis()

Return agent basis

get_result()

Return the current value of x

Returns value of x

Return type numpy.ndarray

iterate_run(*event*)

Run a single iterate of the algorithm

run(*iterations=100*, *verbose=False*, *event=None*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (*int, optional*) – Number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Return type ndarray

Returns the sequence of computed solutions if enable_log=True.

unique_constr(*constraints*)

Remove redundant constraints from given constraint list

Distributed Simplex

class disropt.algorithms.constraintexchange.**DistributedSimplex**(*agent*, *problem_size=None*, *local_indices=None*, *enable_log=False*, *stop_iterations=None*, *big_M=500.0*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Distributed Simplex Algorithm [BuNo11]

This algorithm solves linear programs in standard form When reading the variable agent.problem.constraints, this class only considers equality constraints. Other constraints are discarded.

x
current value of the complete solution

Type numpy.ndarray

J
current value of the cost

Type float

x_basic
current value of the basic solution

Type numpy.ndarray

B
basis associated to the local solution

Type numpy.ndarray

n_constr
number of constraints of the problem

Type tuple

B_neigh
dictionary containing the local bases of (in-)neighbors

Type dict

A_init
initial constraint matrix

Type numpy.ndarray

b_init
initial constraint vector

Type numpy.ndarray

c_init
initial cost vector

Type numpy.ndarray

sequence_x
sequence of solutions

Type numpy.ndarray

sequence_J
sequence of costs

Type numpy.ndarray

Parameters

- **agent** (*Agent*) – agent to execute the algorithm
- **problem_size** (*list*) – total number of variables in the network. Defaults to None. If both problem_size and local_indices is provided, the complete solution vector will be computed.

- **local_indices** (*list*) – indices of the agent's variables in the network, starting from 0. Defaults to None. If both problem_size and local_indices is provided, the complete solution vector will be computed.
- **enable_log** (*bool*) – True to enable log
- **stop_iterations** (*int*) – iterations with constant solution to stop algorithm. Defaults to None (disabled).
- **big_M** (*float*) – cost of big-M variables. Defaults to 500.

check_index_consistency()

Check consistency of local indices, problem_size and constraint matrix

get_basis()

Return current basis

get_result()

Return the current value of the solution

Returns value of primal solution, primal basic solution, dual solution, cost

Return type tuple of nd.ndarray (primal, primal_basic, dual, cost)

initialize()

Evaluate a first solution and basis starting from agent's constraints through the Big-M method

iterate_run(*event*)

Run a single iterate of the algorithm

read_problem_data()

Read local problem data from agent.problem. The data is saved in order to be solved as a standard form problem.

run(*iterations=100*, *verbose=False*, *event=None*, ***kwargs*)

Run the algorithm for a given number of iterations

Parameters

- **iterations** (*int, optional*) – Maximum number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises TypeError – The number of iterations must be an int

Return type ndarray

Returns return a tuple (x, J) with the sequence of solutions and costs if enable_log=True.

Dual Distributed Simplex

class disropt.algorithms.constraintexchange.**DualDistributedSimplex**(*agent*, *num_constraints=None*, *local_indices=None*, *enable_log=False*, *stop_iterations=None*, *big_M=500.0*)

Bases: *disropt.algorithms.constraintexchange.DistributedSimplex*

Distributed Simplex Algorithm on dual problem [BuNo11]

This algorithm solves linear programs of the form This class runs the Distributed Simplex algorithm on the (standard form) dual problem.

Parameters

- **agent** (*Agent*) – agent to execute the algorithm
- **num_constraints** (*list*) – total number of constraints in the network. Defaults to None. If both num_constraints and local_indices is provided, the complete dual solution vector will be computed.
- **local_indices** (*list*) – indices of the agent's constraints in the network, starting from 0. Defaults to None. If both num_constraints and local_indices is provided, the complete dual solution vector will be computed.
- **enable_log** (*bool*) – True to enable log
- **stop_iterations** (*int*) – iterations with constant solution to stop algorithm. Defaults to None (disabled).
- **big_M** (*float*) – cost of big-M variables. Defaults to 500.

`check_index_consistency()`

Check consistency of local indices, num_constraints and constraint matrix

`get_result()`

Return the current value of the solution

Returns value of primal solution, dual basic solution, dual solution, cost

Return type tuple of nd.ndarray (primal, dual_basic, dual, cost)

`read_problem_data()`

Read local problem data from agent.problem. The data is saved in order to be solved as a standard form problem.

References

5.3.6 Miscellaneaous Algorithms

Distributed Logic-And

`class disropt.algorithms.misc.LogicAnd(agent, graph_diameter, flag=False, enable_log=False, **kwargs)`

Bases: *disropt.algorithms.algorithm.Algorithm*

Logic-And algorithm. It can be used for checking in a distributed way if a certain condition (corresponding to flag=True in the algorithm) is satisfied by all the agents in the network. Details can be found in [FaGa19a]

Parameters

- **agent** (*Agent*) – Agent
- **graph_diameter** (*int*) – diameter of the graph representing the network
- **flag** (*bool, optional*) – local flag value. Defaults to False.
- **enable_log** (*bool, optional*) – True for enabling log. Defaults to False.

change_flag(*new_flag*)

Change the local flag

Parameters **new_flag** (bool) – new flag

check_stop()

Check the last row of S

Returns True if last row contains only ones. Meaning that all have the flag True

Return type bool

force_matrix_update()

Force the matrix S to have all ones in the last row

iterate_run()

Run an iterate

matrix_reset()

matrix_update()

Update the matrix S

run(*maximum_iterations=100*, *verbose=False*)

Run the algorithm

Parameters

- **maximum_iterations** (int) – Maximum number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises **TypeError** – maximum iterations must be an int

update_column(*neighbor*, *column*)

Update a column of the matrix corresponding to a neighbor

Parameters

- **neighbor** (Any) – neighbor
- **column** (ndarray) – column value

Raises

- **TypeError** – second argument must be a numpy.ndarray with shape (graph_diameter,)
- **ValueError** – second argument must be a numpy.ndarray with shape (graph_diameter,)

Asynchronous Distributed Logic-And

class disropt.algorithms.misc.**AsynchronousLogicAnd**(*agent*, *graph_diameter*, *flag=False*, *enable_log=False*, ***kwargs*)

Bases: *disropt.algorithms.misc.LogicAnd*

Asyncrhonous Logic-And algorithm. It can be used for checking in a distributed way if a certain condition (corresponding to flag=True in the algorithm) is satisfied by all the agents in the network. Details can be found in [FaGa19a]

Parameters

- **agent** (Agent) – Agent

- **graph_diameter** (*int*) – diameter of the graph representing the network
- **flag** (*bool, optional*) – local flag value. Defaults to False.
- **enable_log** (*bool, optional*) – True for enabling log. Defaults to False.

iterate_run()

Run an iterate

run(*maximum_running_time=1*)

Run the algorithm

Parameters **maximum_running_time** (float) – Maximum running time. Defaults to 1.

Raises TypeError – maximum running time must be a float

Distributed Max-Consensus

class disropt.algorithms.misc.**MaxConsensus**(*agent*, *x0*, *graph_diameter=None*, *enable_log=False*, ***kwargs*)

Bases: *disropt.algorithms.algorithm.Algorithm*

Max-Consensus algorithm. It computes the entry-wise maximum of a numpy array by using only neighboring communication.

Parameters

- **agent** (Agent) – Agent
- **x0** (*np.ndarray*) – local initial condition
- **graph_diameter** (*int, optional*) – diameter of the graph representing the network
- **enable_log** (*bool, optional*) – True to enable log. Defaults to False.

get_result()

Return the value of the solution

iterate_run()

Run an iterate

run(*iterations=100*, *verbose=False*)

Run the algorithm

Parameters

- **iterations** (int) – Maximum number of iterations. Defaults to 100.
- **verbose** (bool) – If True print some information during the evolution of the algorithm. Defaults to False.

Raises TypeError – maximum iterations must be an int

References

Algorithm

class disropt.algorithms.**Algorithm**(*agent*, *enable_log=False*, ***kwargs*)

Bases: object

Algorithm abstract class

Parameters

- **agent** (Agent) – agent to execute the algorithm
- **enable_log** (*bool*) – True for enabling log

agent

agent to execute the algorithm

Type *Agent*

sequence

sequence of data generated by the algorithm

Type numpy.ndarray

enable_log

True for enabling log

Type bool

get_result()

Return the value of the solution

run()

Run the algorithm

5.4 Functions

Functions are used to represent objective functions and constraints in optimization problems (see the tutorial *Objective functions and constraints*).

5.4.1 Abstract Function

AbstractFunction

class disropt.functions.abstract_function.**AbstractFunction**

Bases: object

AbstractFunction class. This should be the parent of all specific (objective) functions.

input_shape

shape of the input of the function

Type tuple

output_shape
shape of the output of the function

Type tuple

differentiable
True if the function is differentiable

Type bool

affine
True if the function is affine

Type bool

quadratic
True if the function is quadratic

Type bool

eval(*x*)
Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

jacobian(*x*, ***kwargs*)
Evaluate the jacobian of the the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

subgradient(*x*, ***kwargs*)
Evaluate the subgradient of the function at a point x

Parameters **x** (ndarray) – input point

Raises **ValueError** – subgradient is defined only for functions with scalar output

Return type ndarray

get_parameters()

hessian(*x*, ***kwargs*)
Evaluate the hessian of the the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

property is_affine

property is_differentiable

property is_quadratic

5.4.2 Basic Functions

Here is the list of the implemented basic mathematical functions.

Variable

class disropt.functions.variable.**Variable**(*n*)

Bases: *disropt.functions.affine_form.AffineForm*

Variable, basic function

$$f(x) = x$$

with $x \in \mathbb{R}^n$

Parameters **n** (*int*) – dimension of the decision variable: (n,1)

Raises **TypeError** – input dimension must be an int

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

AffineForm

class disropt.functions.affine_form.**AffineForm**(*fn*, *A=None*, *b=None*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Makes an affine transformation

$$f(x) = \langle A, x \rangle + b = A^\top x + b$$

with $A \in \mathbb{R}^{n \times m}$, $b \in \mathbb{R}^m$ and $x : \mathbb{R}^n$. It can also be instantiated as:

```
A @ x + b
```

Parameters

- **fn** (AbstractFunction) – input function
- **A** (*numpy.ndarray*) – input matrix
- **b** (*numpy.ndarray*) – input bias

Raises

- **TypeError** – first argument must be a AbstractFunction object
- **TypeError** – second argument must be numpy.ndarray
- **ValueError** – the number of columns of A must be equal to the number of
- **rows of the output of fn** –

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

get_parameters()

QuadraticForm

class disropt.functions.quadratic_form.**QuadraticForm**(*fn*, *P=None*, *q=None*, *r=None*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Quadratic form

$$f(x) = x^\top P x + q^\top x + r$$

with $P \in \mathbb{R}^{n \times n}$, $q \in \mathbb{R}^n$, $r \in \mathbb{R}$ and $x : \mathbb{R}^n$.

Parameters

- **fn** (AbstractFunction) – input function
- **P** (*numpy.ndarray, optional*) – input matrix. Defaults to None (identity).
- **q** (*numpy.ndarray, optional*) – input vector. Defaults to None (zero).
- **r** (*numpy.ndarray, optional*) – input bias. Defaults to None (zero).

Raises

- **TypeError** – First argument must be a AbstractFunction object
- **TypeError** – Second argument must be a numpy.ndarray
- **ValueError** – Input matrix must be a square matrix
- **ValueError** – Dimension mismatch. Input matrix must have shape compliant with the function output shape

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

get_parameters()

Abs

class disropt.functions.abs.**Abs**(*fn*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Absolute value (element-wise)

$$f(x) = |x|$$

with $x : \mathbb{R}^n$.

Parameters **fn** (AbstractFunction) – input function

Raises

- **TypeError** – input must be a function object
- **NotImplementedError** – only 1, 2 and inf norms are currently supported

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Norm

class disropt.functions.norm.**Norm**(*fn*, *order=None*, *axis=None*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Norm of a function (supporte norms are 1, 2, inf)

$$f(x) = \|x\|$$

with $x : \mathbb{R}^n$.

Parameters

- **fn** (AbstractFunction) – input function
- **order** (*int, optional*) – order of the norm. Can be 1, 2 or np.inf. Defaults to 2.

Raises

- **TypeError** – input must be a function object
- **NotImplementedError** – only 1, 2 and inf norms are currently supported

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

SquaredNorm

class disropt.functions.squared_norm.**SquaredNorm**(*fn*, *order=None*, *axis=None*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Squared norm (supporte norms are 1, 2, inf)

$$f(x) = \|x\|^2$$

with $x : \mathbb{R}^n$.

Parameters

- **fn** (AbstractFunction) – input function
- **order** (*int, optional*) – order of the norm. Can be 1, 2 or np.inf. Defaults to 2.

Raises

- **TypeError** – input must be a function object

- **NotImplementedError** – only 1, 2 and inf norms are currently supported

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Log

class disropt.functions.log.**Log**(*fn*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Natural log function (elementwise)

$$f(x) = \log(x)$$

with $x : \mathbb{R}^n$.

Parameters **fn** (AbstractFunction) – input function

Raises **TypeError** – input must be a AbstractFunction object

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Exp

class disropt.functions.exp.**Exp**(*fn*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Exponential function (elementwise)

$$f(x) = e^x$$

with $x : \mathbb{R}^n$.

Parameters **fn** (AbstractFunction) – input function

Raises **TypeError** – input must be a AbstractFunction object

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Logistic

class disropt.functions.logistic.**Logistic**(*fn*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Logistic function (elementwise)

$$f(x) = \log(1 + e^x)$$

with $x : \mathbb{R}^n$.

Parameters **fn** (AbstractFunction) – input function

Raises **TypeError** – input must be a AbstractFunction object

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Min

class disropt.functions.min.**Min**(*f1*, *f2*)

Bases: *disropt.functions.max.Max*

Min function (elementwise)

$$f(x, y) = \min(x, y)$$

with $x, y : \mathbb{R}^n$.

Parameters

- **f1** (AbstractFunction) – input function
- **f2** (AbstractFunction) – input function

Raises

- **ValueError** – input must be a AbstractFunction object
- **ValueError** – sunctions must have the same input/output shapes

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Max

class disropt.functions.max.**Max**(*f1*, *f2*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Max function (elementwise)

$$f(x, y) = \max(x, y)$$

with $x, y : \mathbb{R}^n$.

Parameters

- **f1** (AbstractFunction) – input function
- **f2** (AbstractFunction) – input function

Raises

- **TypeError** – input must be a AbstractFunction object
- **ValueError** – sunctions must have the same input/output shapes

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Square

class disropt.functions.square.**Square**(*fn*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Square function (elementwise)

$$f(x) = x^2$$

with $x : \mathbb{R}^n$.

Parameters **fn** (AbstractFunction) – input function

Raises **TypeError** – input must be a AbstractFunction object

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

Power

class disropt.functions.power.**Power**(*fn*, *exponent*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Power function (elementwise)

$$f(x) = x^{\alpha}$$

with $x : \mathbb{R}^n$, $\alpha : \mathbb{R}$.

Parameters **fn** (AbstractFunction) – input function

Raises **TypeError** – input must be a AbstractFunction object

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

5.4.3 Special Functions

Stochastic Function

class disropt.functions.stochastic_function.**StochasticFunction**(*fn_list*, *probabilities*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Stochastic function

$$f(x) = \mathbb{E}[h(x)]$$

with $x : \mathbb{R}^n$.

The *random_batch* method extract a batch from the function and *batch_subgradient*, *batch_jacobian* and *batch_hessian* methods return a subgradient, jacobian and hessian computed at on the last batch.

Parameters

- **fn_list** (*list*) – list of AbstractFunction objects
- **probabilities** (*list, optional*) – list with the probabilities of drawing each function. Default is None which leads to uniform probabilities

Raises

- **TypeError** – fn_list input must be a list of functions
- **ValueError** – All functions must have the same input/output shape
- **TypeError** – probabilities argument must be a list of floats
- **ValueError** – inputs must have the same lenght
- **ValueError** – provided probabilities must sum to 1
- **NotImplementedError** – only 1, 2 and inf norms are currently supported

eval(*x*)

Evaluate the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

jacobian(*x*, ***kwargs*)

Evaluate the jacobian of the the function at a point x

Parameters **x** (ndarray) – input point

Return type ndarray

subgradient(*x*, ***kwargs*)

Evaluate the subgradient of the function at a point x

Parameters **x** (ndarray) – input point

Raises **ValueError** – subgradient is defined only for functions with scalar output

Return type ndarray

batch_hessian(*x*)

evaluate the hessian on the current batch

Parameters **x** (*np.ndarray*) – point

Returns hessian

Return type numpy.ndarray

batch_jacobian(*x*)

evaluate the jacobian on the current batch

Parameters **x** (*np.ndarray*) – point

Returns jacobian

Return type numpy.ndarray

batch_subgradient(*x*)

evaluate the subgradient on the current batch

Parameters **x** (*np.ndarray*) – point

Raises **ValueError** – Only functions with scalar output have a subgradient

Returns subgradient

Return type numpy.ndarray

random_batch(*batch_size=1*)

generate a random batch from the function.

Parameters **batch_size** (*int, optional*) – batch size. Defaults to 1.

Function With Extended Variable

class disropt.functions.extended_function.**ExtendedFunction**(*fn*, *n_var=1*, *axis=- 1*, *pos=0*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Function with extended variable

$$f(x, y) = x$$

with $x \in \mathbb{R}^n, y \in \mathbb{R}^m$

Parameters

- **fn** (AbstractFunction) – input function
- **n_var** (int) – number of additional variables. Defaults to 1
- **axis** (int) – axis along which the additional variables are appended. Defaults to -1 (the last valid one)
- **pos** (int) – position index of the old variable vector. Defaults to 0

Raises

- **TypeError** – fn must be a AbstractFunction
- **TypeError** – n_var must be a positive int
- **TypeError** – axis must be int

5.4.4 SubmodularFn

class disropt.functions.submodular_func.**SubmodularFn**(*input_shape*)

Bases: *disropt.functions.abstract_function.AbstractFunction*

Submodular AbstractFunction abstract class

V

ground set

input_shape

cardinality of the ground set

Parameters **input_shape** – cardinality of the ground set

Raises **ValueError** – input_shape must be an integer positive number

greedy_polyhedron(*self*, *w*)

Greedy algorithm for finding a maximizer x of

$$\max_{x \in B(F)} w^\top x \tag{5.3}$$

where $B(F)$ is the base polyhedron associated to the submodular function F

Parameters **w** – cost direction

Returns maximizer of (5.3)

Return type numpy.ndarray

Raises **ValueError** – Input must be a numpy.ndarray with input_shape elements

subgradient(*self*, *x*)

Evaluate a subgradient of the Lovasz extension of the submodular function at x

Parameters **x** – vector

Returns subgradient of the Lovasz extension of F at x

Return type numpy.ndarray

Raises **ValueError** – Input must be a numpy.ndarray with input_shape elements

blocksubgradient(*x*, *block*)

Evaluate a subgradient of the Lovasz extension of the submodular function at x

Parameters

- **x** – vector
- **block** – vector

Returns block subgradient of the Lovasz extension of F at x

Return type numpy.ndarray

Raises **ValueError** – Input must be a numpy.ndarray with input_shape elements

eval(*set*)

Evaluate the submodular function at a given set

Parameters **set** (*numpy.ndarray(, dtype='int')*) – vector

5.5 Constraints

5.5.1 Generic constraints

AbstractConstraint (Abstract class)

class disropt.constraints.constraints.**AbstractConstraint**

Bases: object

Abstract class for expressing constraints

eval()

Constraint

class disropt.constraints.constraints.**Constraint**(*fn*, *sign='=='*)

Bases: *disropt.constraints.constraints.AbstractConstraint*

Constraint build from a AbstractFunction object. Constraints are represented in the canonical forms $f(x) = 0$ and $f(x) \leq 0$.

Parameters

- **fn** (AbstractFunction) – constraint function
- **sign** (*bool*) – type of constraint: "==", "<=" or ">="

fn

constraint function

Type *AbstractFunction*

sign

type of constraint: "==", "<=" or ">="

Type bool

input_shape

input space dimensions

Type tuple

output_shape

output space dimensions

Type tuple

eval(*x*)

Evaluate the constraint function at a point x

Parameters **x** (ndarray) – input point

Return type bool

property function

get_parameters()

Return the parameters of the function if it is affine or quadratic

Returns A, b for affine constraints, P, q, r for quadratic

Return type tuple

property is_affine

Return true if the function is affine.

Returns true if the function is affine

Return type bool

property is_equality

property is_inequality

property is_quadratic

Return true if the function is affine.

Returns true if the function is affine

Return type bool

projection(*x*)

Compute the projection of a point onto the set defined by the constraint. The constraint should be convex.

Parameters **x** (ndarray) – point to be projected

Returns projected point

Return type numpy.ndarray

ExtendedConstraint

class disropt.constraints.extended_constraint.**ExtendedConstraint**(*fn*, *sign='=='*)

Bases: *disropt.constraints.constraints.Constraint*

Constraint with extended variable

Parameters

- **constr** (*Constraint or list of Constraint*) – original constraint(s)
- **n_var** – number of additional variables. Defaults to 1
- **axis** – axis along which the additional variables are appended. Defaults to -1 (the last valid one)
- **pos** – position index of the old variable vector. Defaults to 0

Raises

- **TypeError** – fn must be a Constraint object or a list of Constraint objects
- **TypeError** – n_var must be a positive int
- **TypeError** – axis must be int

5.5.2 Sets (with project method)

AbstractSet

class disropt.constraints.projection_sets.**AbstractSet**

Bases: object

Abstract constraint set

projection(*x*)

Project a point onto the set

to_constraints()

Convert the set in a list of Constraints

Box

class disropt.constraints.projection_sets.**Box**(*lower_bound=None*, *upper_bound=None*)

Bases: *disropt.constraints.projection_sets.AbstractSet*

Box set

$X = \{x \mid l \leq x \leq u\}$ with $l, x, u \in \mathbb{R}^n$

Parameters

- **lower_bound** (*numpy.ndarray, optional*) – array with lower bounds for each axis. Defaults to None.
- **upper_bound** (*numpy.ndarray, optional*) – array with upper bounds for each axis. Defaults to None.

lower_bound
lower bounds

Type numpy.ndarray

upper_bound
upper bounds

Type numpy.ndarray

input_shape
space dimensions

Type tuple

intersection(*box*)
Compute the intersection with another box

Parameters **box** (Box) – box to compute the intersection with

Raises

- **ValueError** – Only intersection with another box is supported
- **ValueError** – The two boxex must have the same input_shape

projection(*x*)
Project a point onto the box

Parameters **x** (*numpy.ndarray*) – Point to be projected

Returns projected point

Return type numpy.ndarray

to_constraints()
Convert the set in a list of Constraints

Strip

class disropt.constraints.projection_sets.**Strip**(*regressor*, *shift*, *width*)
Bases: *disropt.constraints.projection_sets.AbstractSet*

Strip set

$X = \{x \mid -w <= |a^\top x - s| <= w\}$ with $a, x \in \mathbb{R}^n$ and $w \in \mathbb{R}$

Parameters

- **regressor** (*numpy.ndarray*) – regressor of the strip
- **shift** (*float*) – shift from the origin
- **width** (*float*) – width of the strip

regressor
regressor of the strip

Type numpy.ndarray

shift
shift from the origin

Type float

upper
upper border (shift + half width)

Type float

lower
lower border (shift - half width)

Type float

input_shape
space dimensions

Type tuple

intersection(*strip*)
Compute the intersection with another strip

Parameters **strip** (Strip) – strip to compute the intersection with

Raises

- **ValueError** – Only intersection with another strip is supported
- **ValueError** – The two strips must have the same regressor

projection(*x*)
Project a point onto the strip

Parameters **x** (*numpy.ndarray*) – Point to be projected

Returns projected point

Return type numpy.ndarray

to_constraints()
Convert the set in a list of Constraints

Circle

class disropt.constraints.projection_sets.**Circle**(*center*, *radius*)
Bases: *disropt.constraints.projection_sets.AbstractSet*

Circle set

$X = \{x \mid \|x - c\| <= r\}$ with $x, c \in \mathbb{R}^n$ and $r \in \mathbb{R}$

Parameters

- **center** (*numpy.ndarray*) – center of the circle
- **radius** (*float*) – radius of the circle

center
center of the circle

Type numpy.ndarray

radius
radius of the circle

Type float

input_shape
space dimensions

Type tuple

intersection(*circle*)
Compute the intersection with another circle

Parameters **circle** (Circle) – circle to compute the intersection with

Raises

- **ValueError** – Only intersection with another circle is supported
- **ValueError** – The two circles must have the same center

projection(*x*)
Project a point onto the circle

Parameters **x** (*numpy.ndarray*) – Point to be projected

Returns projected point

Return type numpy.ndarray

to_constraints()
Convert the set in a list of Constraints

CircularSector

class disropt.constraints.projection_sets.**CircularSector**(*vertex*, *angle*, *radius*, *width*)
Bases: *disropt.constraints.projection_sets.AbstractSet*

Circular sector set.

Parameters

- **vertex** (*numpy.ndarray*) – vertex of the circular sector
- **angle** (*float*) – direction of the circular sector (in rad)
- **radius** (*float*) – radius of the circular sector
- **width** (*float*) – width of the circular sector (in rad)

vertex
vertex of the circular sector

Type numpy.ndarray

angle
direction of the circular sector (in rad)

Type float

radius

radius of the circular sector

Type float

h_angle

left border (from the vertex pov) of the circular sector (in rad)

Type float

l_angle

right border (from the vertex pov) of the circular sector (in rad)

Type float

input_shape

space dimensions

Type tuple

intersection(*circular_sector*)

Compute the intersection with another circular sector

Parameters **circular_sector** (CircularSector) – circula sector to compute the intersection with

Raises

- **ValueError** – Only intersection with another circular_sector is supported
- **ValueError** – The two circular_sector must have the same vertex

projection(x)

Project a point onto the ciruclar sector

Parameters **x** (*numpy.ndarray*) – Point to be projected

Returns projected point

Return type numpy.ndarray

to_constraints()

Convert the set in a list of Constraints

5.6 Problem Classes

5.6.1 Problem

class disropt.problems.**Problem**(*objective_function=None*, *constraints=None*, *force_general_problem=False*, ***kwargs*)

Bases: object

A generic optimization problem.

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } g(x) \leq 0 \\ &\qquad\qquad h(x) = 0 \end{aligned}$$

Parameters

- **objective_function** (AbstractFunction, *optional*) – objective function. Defaults to None.
- **constraints** (*list*, *optional*) – constraints. Defaults to None.

objective_function

Objective function to be minimized

Type Function

constraints

constraints

Type list

input_shape

dimension of optimization variable

Type tuple

output_shape

output shape

Type tuple

add_constraint(*fn*)

Add a new constraint

Parameters **fn** (Union[*AbstractSet*, *Constraint*]) – new constraint

Raises **TypeError** – constraints must be AbstractSet or Constraint

project_on_constraint_set(*x*)

Compute the projection of a point onto the constraint set of the problem

Parameters **x** (ndarray) – point to project

Returns projected point

Return type numpy.ndarray

set_objective_function(*fn*)

Set the objective function

Parameters **fn** (*AbstractFunction*) – objective function

Raises **TypeError** – input must be a AbstractFunction object

solve(*solver='cvxpy'*, *return_only_solution=True*)

Solve the problem

Returns solution

Return type numpy.ndarray

5.6.2 LinearProblem

class disropt.problems.**LinearProblem**(*objective_function*, *constraints=None*, ***kwargs*)

Bases: *disropt.problems.problem.Problem*

Solve a Linear programming problem defined as:

$$\begin{aligned} &\text{minimize } c^\top x \\ &\text{subject to } Gx \leq h \\ &\qquad\qquad Ax = b \end{aligned}$$

set_objective_function(*objective_function*)

set the objective function

Parameters **objective_function** (AffineForm) – objective function

Raises **TypeError** – Objective function must be a AffineForm with output_shape=(1,1)

solve(*initial_value=None*, *solver='glpk'*, *return_only_solution=True*)

Solve the problem

Parameters

- **initial_value** (*numpy.ndarray), optional*) – Initial value for warm start. Defaults to None.
- **solver** (*str, optional*) – Solver to use [‘glpk’, ‘cvxopt’]. Defaults to ‘glpk’.

Raises **ValueError** – Unsupported solver

Returns solution

Return type numpy.ndarray

5.6.3 MixedIntegerLinearProblem

class disropt.problems.**MixedIntegerLinearProblem**(*objective_function*, *constraints=None*, *integer_vars=None*, *binary_vars=None*, ***kwargs*)

Bases: *disropt.problems.linear_problem.LinearProblem*

Solve a Mixed-Integer Linear programming problem defined as:

$$\begin{aligned} &\text{minimize } c^\top x \\ &\text{subject to } Gx \leq h \\ &\qquad\qquad Ax = b \\ &\qquad\qquad x_k \in \mathbb{Z}, \forall k \in I \\ &\qquad\qquad x_k \in \{0,1\}, \forall k \in B \end{aligned}$$

where I is the set of integer variable indexes and B is the set of binary variable indexes.

solve(*initial_value=None*, *solver='glpk'*, *return_only_solution=True*)

Solve the problem

Parameters

- **initial_value** (*numpy.ndarray), optional*) – Initial value for warm start. Defaults to None. Not available in GLPK

- **solver** (*str, optional*) – Solver to use ['glpk', 'gurobi']. Defaults to 'glpk'.

Raises ValueError – Unsupported solver

Returns solution

Return type numpy.ndarray

5.6.4 ConvexifiedMILP

class disropt.problems.**ConvexifiedMILP**(*objective_function*, *y*, *A*, *constraints=None*, *integer_vars=None*, *binary_vars=None*, ***kwargs*)

Bases: *disropt.problems.milp.MixedIntegerLinearProblem*

Solve a convexified Mixed-Integer Linear Problem of the form:

$$\begin{aligned} &\text{minimize } c^\top x + M\rho \\ &\text{subject to } x \in \text{conv}(X),\ \rho \geq 0 \\ &Ax \leq y + \rho \mathbf{1} \end{aligned}$$

where the set X is a compact mixed-integer polyhedral set defined by equality and inequality constraints. Moreover, ρ is a scalar, $y \in \mathbb{R}^m$ is a vector and $A \in \mathbb{R}^{m \times n}$ is a constraint matrix.

solve(*M*, *milp_solver=None*, *initial_dual_solution=None*, *return_only_solution=True*, *y=None*, *max_cutting_planes=None*, *cutting_planes=None*, *threshold_convergence=1e-08*, *max_iterations=1000*)

Solve the problem using a custom dual cutting-plane algorithm

Parameters

- **M** (*float*) – value of the parameter M
- **milp_solver** (*str, optional*) – MILP solver to use. Defaults to None (use default solver).
- **initial_dual_solution** (*numpy.ndarray, optional*) – Initial dual value for warm start. Defaults to None.
- **return_only_solution** (*bool, optional*) – if True, returns only solution, otherwise returns more information. Defaults to True.
- **y** (*np.ndarray, optional*) – value to override the current y. Defaults to None (keep current value).
- **max_cutting_planes** (*int, optional*) – maximum number of stored cutting planes. Defaults to None (disabled).
- **cutting_planes** (*numpy.ndarray, optional*) – cutting planes for warm start previously returned by this function. Defaults to None.
- **threshold_convergence** (*float, optional*) – threshold for convergence detection. Defaults to 1e-8.
- **max_iterations** (*int, optional*) – maximum number of iterations performed by algorithm. Defaults to 1e3.

Returns primal solution tuple (x, rho) if return_only_solution = True, otherwise (primal solution tuple, optimal cost, dual solution, cutting planes, number of iterations)

Return type tuple

5.6.5 QuadraticProblem

class disropt.problems.**QuadraticProblem**(*objective_function*, *constraints=None*, *is_pos_def=True*, ***kwargs*)

Bases: *disropt.problems.problem.Problem*

Solve a Quadratic programming problem defined as:

$$\begin{aligned} &\text{minimize } x^\top P x + q^\top x + r \\ &\text{subject to } Gx \leq h \\ &\qquad\quad Ax = b \end{aligned}$$

Quadratic problems are currently solved by using CVXOPT or OSQP https://osqp.org.

Parameters

- **objective_function** (QuadraticForm) – objective function
- **constraints** (*list, optional*) – list of constraints. Defaults to None.
- **is_pos_def** (*bool*) – True if P is (semi)positive definite. Defaults to True.

set_constraints(*constraints*)

Set the constraints

Parameters **constraints** (*list*) – list of constraints

Raises **TypeError** – a list of affine Constraints must be provided

set_objective_function(*objective_function*)

set the objective function

Parameters **objective_function** (QuadraticForm) – objective function

Raises **TypeError** – Objective function must be a QuadraticForm

solve(*initial_value=None*, *solver='osqp'*, *return_only_solution=True*)

Solve the problem

Parameters

- **initial_value** (*numpy.ndarray), optional*) – Initial value for warm start. Defaults to None.
- **solver** (*str, optional*) – Solver to use ('osqp' or 'cvxopt'). Defaults to 'osqp'.

Raises **ValueError** – Unsupported solver, only 'osqp' and 'cvxopt' are currently supported

Returns solution

Return type numpy.ndarray

5.6.6 ProjectionProblem

class disropt.problems.**ProjectionProblem**(**args*, ***kwargs*)

Bases: *disropt.problems.problem.Problem*

Computes the projection of a point onto some constraints, i.e., it solves

$$\begin{aligned} &\text{minimize } \frac{1}{2}\|x - p\|^2 \\ &\text{subject to } f_k(x) \leq 0,\ k = 1, \ldots \end{aligned}$$

Parameters

- **constraints_list** (*list*) – list of constraints
- **point** (*numpy.ndarray*) – point p to project

solve()

solve the problem

Returns solution

Return type numpy.ndarray

5.6.7 ConstraintCoupledProblem

class disropt.problems.**ConstraintCoupledProblem**(*objective_function=None*, *constraints=None*, *coupling_function=None*, ***kwargs*)

Bases: *disropt.problems.problem.Problem*

A local part of a constraint-coupled problem.

Parameters

- **objective_function** (*AbstractFunction, optional*) – Local objective function. Defaults to None.
- **constraints** (*list, optional*) – Local constraints. Defaults to None.
- **coupling_function** (*AbstractFunction, optional*) – Local function contributing to coupling constraints. Defaults to None.

objective_function

Objective function to be minimized

Type Function

constraints

Local constraints

Type *AbstractSet* or *Constraint*

coupling_function

Local function contributing to coupling constraints

Type Function

set_coupling_function(*fn*)

Set the coupling constraint function

Parameters **fn** (*AbstractFunction*) – coupling constraint function

Raises **TypeError** – input must be a AbstractFunction

5.6.8 ConstraintCoupledMILP

class disropt.problems.**ConstraintCoupledMILP**(*objective_function*, *coupling_function*, *constraints=None*, *integer_vars=None*, *binary_vars=None*, ***kwargs*)

Bases: *disropt.problems.constraint_coupled_problem.ConstraintCoupledProblem*, *disropt.problems.milp.MixedIntegerLinearProblem*

5.7 Utilities

5.7.1 Graph generation

MPIgraph

class disropt.utils.graph_constructor.**MPIgraph**(*graph_type=None*, *in_weight_matrix_type=None*, *out_weight_matrix_type=None*, ***kwargs*)

Bases: object

Create a graph on the network

Parameters

- **graph_type** (*str, optional*) – type of graph ('complete', 'random_binomial'). Defaults to None (complete).
- **in_weight_matrix_type** (*str, optional*) – type of matrix describing in-neighbors weights ('metropolis', 'row_stochastic', 'column_stochastic'). Defaults to None (metropolis).
- **out_weight_matrix_type** (*str, optional*) – type of matrix describing out-neighbors weights ('metropolis', 'row_stochastic', 'column_stochastic'). Defaults to None (metropolis).

get_local_info()

return the local info available at the agent

Returns local_rank, in_neighbors, out_neighbors, in_weights, out_weights,

Return type tuple

ring_graph

disropt.utils.graph_constructor.**ring_graph**(*N*, *link_type='undirected'*)

construct a ring graph

Parameters

- **N** (*int*) – number of agents
- **link_type** (*str, optional*) – 'directed' or 'undirected'. Defaults to 'undirected'.

Returns adjacency matrix

Return type numpy.ndarray

binomial_random_graph

disropt.utils.graph_constructor.**binomial_random_graph**(*N*, *p=None*, *seed=None*, *link_type='undirected'*)

construct a random binomial graph

Parameters

- **N** (*int*) – number of agents
- **p** (*float, optional*) – link probability. Defaults to None (=1).
- **seed** (*int, optional*) – [description]. Defaults to None (=1).
- **link_type** (*str, optional*) – 'directed' or 'undirected'. Defaults to 'undirected'.

Returns adjacency matrix

Return type numpy.ndarray

binomial_random_graph_sequence

disropt.utils.graph_constructor.**binomial_random_graph_sequence**(*under_adj*, *n_graphs*, *p=0.1*, *period=None*, *seed=None*, *link_type='undirected'*)

Construct a sequence of random binomial graphs starting from a given underlying graph

Parameters

- **under_adj** (*2D numpy.ndarray*) – Adjacency matrix of underlying graph.
- **n_graphs** (*int*) – number of graphs in the returned sequence.
- **p** (*float or 2D numpy.ndarray, optional*) – link probability. Defaults to None (=0.1).
- **period** (*int, optional*) – T-connectivity period of the returned sequence (obtained artificially with cycles). Defaults to None (no T-connectivity).
- **seed** (*int, optional*) – [description]. Defaults to None (=1).
- **link_type** (*str, optional*) – 'directed' or 'undirected'. Defaults to 'undirected'.

Returns sequence of adjacency matrices

Return type tuple(numpy.ndarray)

5.7.2 Weighted adjacency matrices

metropolis_hastings

disropt.utils.graph_constructor.**metropolis_hastings**(*Adj*, *link_type='undirected'*)

Construct a weight matrix using the Metropolis-Hastings method

Parameters **Adj** (*numpy.ndarray*) – Adjacency matrix

Returns weighted adjacency matrix

Return type numpy.ndarray

row_stochastic_matrix

disropt.utils.graph_constructor.**row_stochastic_matrix**(*Adj*, *weights_type='uniform'*)

Construct a row-stochastic weighted adjacency matrix

Parameters **Adj** (*numpy.ndarray*) – Adjacency matrix

Returns weighted adjacency matrix

Return type numpy.ndarray

column_stochastic_matrix

disropt.utils.graph_constructor.**column_stochastic_matrix**(*Adj*, *weights_type='uniform'*)

Construct a column-stochastic weighted adjacency matrix

Parameters **Adj** (*numpy.ndarray*) – Adjacency matrix

Returns weighted adjacency matrix

Return type numpy.ndarray

5.7.3 Matrix properties

is_pos_def

disropt.utils.utilities.**is_pos_def**(*P*)

check if a matrix is positive definite

Parameters **P** (*numpy.ndarray*) – matrix

Return type bool

is_semi_pos_def

disropt.utils.utilities.**is_semi_pos_def**(*P*)

check if a matrix is positive semi-definite

Parameters **P** (*numpy.ndarray*) – matrix

Return type bool

check_symmetric

disropt.utils.utilities.**check_symmetric**(*A*, *rtol=1e-05*, *atol=1e-08*)

check if a matrix is symetric

Parameters

- **A** (*numpy.ndarray*) – matrix
- **rtol** (*float*) – Defaults to 1e-05.
- **atol** (*float*) – Defaults to 1e-08.

Return type bool

5.7.4 Linear Programming

generate_LP

disropt.utils.LP_utils.**generate_LP**(*n_var*, *n_constr*, *radius*, *direction='min'*, *constr_form='ineq'*)

Generate a feasible and not unbounded Linear Program and return problem data (cost, constraints, solution) TODO add reference

Parameters

- **n_var** (*int*) – number of optimization variables
- **n_constr** (*int*) – number of constraints
- **radius** (*float*) – size of feasible set (for constr_form = 'ineq'), size of dual feasible set (for constr_form = 'eq')
- **direction** (*str, optional*) – optimization direction - either 'max' (for maximization) or 'min' (for minimization). Defaults to 'min'
- **constr_form** (*str, optional*) – form of constraints - either 'ineq' (for inequality: $Ax \leq b$) or 'eq' (for standard form: $Ax = b, x \geq 0$). Defaults to 'ineq'

Returns

- **c** (*numpy.ndarray*): cost vector
- **A** (*numpy.ndarray*): constraint matrix
- **b** (*numpy.ndarray*): constraint vector (right-hand side)
- **solution** (*numpy.ndarray*): optimal solution of problem

Return type tuple

CHAPTER

SIX

ADVANCED FEATURES

TODO explain how the package can be extended

6.1 Optimization Problems

The `Problem` class allows one to define optimization problems of various types. Consider the following problem:

$$\begin{aligned} &\text{minimize } \|A^\top x - b\| \\ &\text{subject to } x \geq 0 \end{aligned}$$

with $x \in \mathbb{R}^4$. We can define it as:

```
import numpy as np
from disropt.problems import Problem
from disropt.functions import Variable, Norm

x = Variable(4)
A = np.random.randn(n, n)
b = np.random.randn(n, 1)

obj = Norm(A @ x - b)
constr = x >= 0

pb = Problem(objective_function = obj, constraints = constr)
```

If the problem is convex, it can be solved as:

```
solution = pb.solve()
```

Generic (convex) nonlinear problems of the form

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } g(x) \leq 0 \\ &\qquad\qquad h(x) = 0 \end{aligned}$$

are solved through the cvxpy solver (when possible), or with the cvxopt solver, while more structured problems (LPs and QPs) can be solved through other solvers (osqp and glpk). The integration with other solvers will be provided in future releases. LPs and QPs can be directly defined through specific classes (`LinearProblem` and `QuadraticProblem`). However, the `Problem` class is capable to recognize LPs and QPs, which are automatically converted into the appropriate format.

6.1.1 Projection onto the constraints set

Projecting a point onto the constraints set of a problem is often required in distributed optimization algorithms. The method `project_on_constraint_set` is available to do this:

```
projected_point = pb.project_on_constraint_set(pt)
```

6.2 Implementing custom functions

Custom functions can be easily implemented and integrated with alredy defined functions. They can be built on the `AbstractFunction` class, by overloading the `eval` method. Subgradients, jacobians and hessians are usually automatically computed through autograd. If they cannot be computed, then the `_alternative_jacobian` and `_alternative_hessian` method must be implemented too.

WARNING: jacobian of custom functions should be implemented by using the numerator layout convention.

CHAPTER

SEVEN

ACKNOWLEDGEMENTS

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http://www.opt4smart.eu

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